REFERENCE PAPERS ON MARKET ORIENTED MANAGEMENT SYSTEMS

225-66

NOV 2 1966

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Prepared for the Third M.I.T. Industrial Liaison Symposium, November 3, 1966

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1. The Evolution of Market Oriented Management Information Systems -- Arnold E. Amstutz

   This presentation examines characteristics of successful management information systems in use by operating and policy managers concerned with the marketing function. It focuses on the attributes and impact of representative systems and the characteristics of managers responsible for their successful implementation. Bases for the evaluation of information systems will be proposed. Representative systems performing retrieval, monitor, advisory, and decision functions are examined. Management implications of on-line, real time, and simulation based system structures are also discussed.

   See paper entitled, "The Marketing Executive and Management Information Systems".

2. Total Environment Management Control Systems -- Arnold E. Amstutz

   This presentation focuses on the development, testing, and implementation of large scale computerized micro-analytic simulation of market behavior. The characteristics of management information systems founded on the use of this technique are discussed using examples from systems in use. Problems associated with model conception and specification, function verification, and system and sub-system testing and validation are discussed with reference to operating systems. Sample simulation runs based on test markets are evaluated and representative management uses of the system are discussed.

   See paper entitled, "Simulation Techniques in the Analysis of Marketing Strategy".

3. Interactive Operating Systems -- A Demonstration

   A Geographic Model of an Urban Automobile Market -- John D. C. Little and Theodore E. Hlavac

   A person chooses a store to shop at partly on the basis of the difficulty of getting there. An understanding of the relationships involved is important in marketing strategy, particularly for site selection purposes. We study an urban new-car market. As a measure of shopping difficulty, the distance from a person's residence to the car dealer is used. Assumptions about buyer behavior lead to a model of competitive interaction among dealers and car makes. The model is fitted to three months of sales data for metropolitan Chicago. An interactive computer system has been programmed to make the model available for on-line investigation of a variety of site selection and inter-brand competition questions. For example, a user can add, eliminate, or move a dealer and then ask for the model-predicted changes in the sales and penetration of any dealer or make.

   See paper entitled, "A Geographic Model of an Urban Automobile Market".

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Computer Aided Preparation of Magazine Advertisement Formats -- Daniel S. Diamond

While most attempts toward quantification in advertising have involved budgets, media selection, and media scheduling, the present presentation deals directly with magazine advertisement design. It is first proposed that before an advertisement can have any effect on the consumer, it must attract his attention. Readership is suggested as a measure of the attention-getting power of an advertisement. Next, a set of six Starch Magazine Advertisement Readership models is constructed by means of multiple regression analysis on 1,070 advertisements appearing in LIFE Magazine. The independent variables are predominantly advertisement format descriptors, such as size, number of colors, and position in magazine -- twelve in all. Finally, a conversational computer program is developed which requests from its user a readership objective function (a function of the six Starch readership scores for the advertisement and its cost), format restrictions, and a budget constraint. The program then prepares that advertisement format, which while conforming to all restrictions, maximizes the objective function.

See paper entitled, "Computer Aided Preparation of Magazine Advertisement Formats".

4. Adaptive Management Control Systems -- John D. C. Little

Companies try to conduct their marketing operations so as to respond to changing market conditions. A model of such a process is studied for the case of setting promotion rate. Company sales are functions of promotional spending, but the relationship changes with time. An adaptive system is devised that works as follows: Information about sales response is collected by performing an experiment. The experimental results are used to update a sales response model. Promotion rate is chosen to maximize expected profit in the next time period. The cycle is repeated. In designing the experiment, sample size is chosen to minimize the cost of imperfect information plus the cost of experimentation.

The model employs a quadratic sales response function with a parameter that changes according to a random walk. The optimal adaptive system turns out to involve exponential smoothing of the experimental results. A numerical example is studied analytically and by simulation. The adaptive system is found to work better than various other policies. In a sensitivity analysis, an adaptive system derived for one underlying model of the market is found to perform well even when certain other models actually apply.

See paper entitled, "A Model of Adaptive Control of Promotional Spending".

THE MARKETING EXECUTIVE AND MANAGEMENT INFORMATION SYSTEMS

213-66

Arnold E. Amstutz

THE MARKETING EXECUTIVE AND MANAGEMENT INFORMATION SYSTEMS*

Arnold E. Amstutz

Introduction

This paper is concerned with the present state of the art in market oriented management information and control systems and the impact of these systems on policy and operating management. As such, this discussion will be narrowly focused. Normative questions of what management should or should not expect or obtain from an information system will not be considered.

Objectives

The two major objectives of this paper may be simply stated as:

1. To identify relevant characteristics of successfully implemented management information and control systems.
2. To evaluate the impact of representative systems on the managements they were designed to serve.

The Process of System Evolution

It is seldom possible to impose a pre-packaged information system on a management group. There are no generalized management information systems. Each company's management has unique information requirements; a unique perspective on the environment within and outside of their firm; unique priorities; and a style of management which is the unique product of the particular personalities making up the management group. Successful system development is a matter of evolution. Effective management systems evolve over time as management and system specialists learn to communicate, structure problems, and achieve a joint focus on increasingly broad information needs.

About the Author. Arnold E. Amstutz is Assistant Professor of Management at the Sloan School of Management, Massachusetts Institute of Technology. He is engaged in research and consulting work relating to applications of computer technology in marketing management and is the author of several articles in this area.

*A talk presented to the American Marketing Association Meeting in Bloomington, Indiana, September 1, 1966.

The process of designing and developing a management information system to meet the requirements of a particular management's decision style makes significant demands on management time and thought. If the resulting product is to be compatible with management's perspective, priorities, and systems of measures, management's models of the decision environment must be made explicit and used as the basis of system design. In addition, management must evaluate the implications of alternative system structures in terms of criteria which only they can supply. Intelligent choice between alternatives must be based on management understanding of the implications of available structures. In order to achieve understanding the manager must take the time necessary to become familiar with the management implication of basic system design concepts.

Just as there is no single generic management information system, there is no single payoff associated with all systems. It may be argued, in fact, that the benefits gained from a particular system are largely determined by the demands which management makes of the system and the capabilities established in the course of system design.

Dimensions of Evaluation

No two managements have the same information needs. As indicated earlier, the characteristics of systems developed for different managements are as different as the managers' personalities, the problems they face, and their approach to the complex art of management. Despite these differences it is possible to identify sets of dimensions for use in evaluating specific systems and isolating similarities and differences between systems. Figures 1 and 2 illustrate four such dimensions.

Information Recency

The first dimension, information recency, refers to the time lapse between occurrence of an event in the environment and inclusion of data describing that

event in the system. This may range from several weeks in the case of certain market developments to a few hours or minutes for automated inventory control.

Information Aggregation

The second dimension, information aggregation, describes the detail with which information is maintained in system data files. Inventory control systems in which information regarding product components or sub-assemblies is maintained at the item level are representative of relatively disaggregated (micro) data maintenance while industry market share statistics of the type developed through trade associations are representative of highly aggregate (macro) measures.

As illustrated in Figure 1, there is normally a relationship between level of aggregation and the time delay involved in incorporating associated data in the system.

Analytical Sophistication

The third dimension, analytical sophistication, refers to the sophistication of models or structure encompassed by the system. As illustrated in Figure 2, the lowest level of analytical sophistication is that required to identify a particular file and record. At this level it is only necessary for the computer to retrieve the specified record and display the information which it contains. The second level of analytic sophistication involves aggregation -- gathering together numbers from within one or more records to produce a total or sub-total. At the third level the computer may be programmed to perform arithmetic averaging or to compute differences. The fourth level, logical analysis, introduces the use of classification schemes through which various types of data are aggregated within sub-sets or conditionally segmented.

At the fifth level of analytical sophistication statistical analyses may be employed to develop extrapolations from historic data, statistical best estimates, analyses of variance, or trend estimates.

The term learning is used in Figure 2 to indicate adaptive system processes through which the computer is programmed to modify parameter values or model

structures on the basis of experience (data inputs received) over time.

At the most advanced level of analytic sophistication, simulation, models on which the system is based constitute an artificial environment paralleling real world markets referenced by the information systems. The managers' perception of the environment has been explicitly modeled to a sufficient extent and detail to justify the assumption that the models making up the simulated environment duplicate in all relevant aspects the response pattern of the real world environment monitored through the information system. Inputs to the information system are directly related to process variables in the simulation models. At this stage in development the system provides management with the capability of testing proposed policy and strategies in the simulated environment; choosing between alternatives on the basis of resulting output; implementing the policies in the real world environment; and evaluating the effectiveness of implemented plans through the information system. The manager references the simulated environment to ask "What if"? and the information system monitoring the real world environment to determine "What is"?.

Computer Authority

The final dimensions of system evaluation, authority delegated to the computer, is closely associated with the system's analytic sophistication. Management is more willing to delegate authority to sophisticated systems and, conversely, as management places greater demands on an information system, a greater level of analytic sophistication must be embodied in the system structure.

At the lowest level management may delegate to the computer authority to retrieve information from specified records and files -- entrust to the computer system processes associated with identification and retrieval. Once a retrieval capability has been established it is usually a short step to the next level of computer authority. Recognizing that the computer has access to all records in the file, management concludes that while the computer is "looking at" the contents of each record it might as well check the reasonableness of record

content to insure against gross clerical errors. At this stage the computer is delegated a supervisory function checking on human personnel responsible for input.

As management comes to accept computer review for purposes of error detection, they normally begin to think in terms of other functions which the computer could perform "while looking at all those records". It follows quite naturally to have the computer perform additional analyses on records which it is reviewing and refer for further review and action situations meeting criteria established by management.

Management frequently finds that certain classes of monitor output are consistently subjected to additional analyses to determine whether or not action is warranted. In such situations it is natural to suggest that the computer be programmed to perform the additional calculations in order to add a recommendation for action to the monitor report.

As management gains experience with computer based recommendations they may find that in most situations they are able to implement computer recommendations without further investigation. Criteria may be modified to isolate non-typical cases requiring additional review. The computer is then given authority to take action on the remaining cases in which its recommendations are a valid basis for action.

The hierarchy of Figure 2 suggests that delegation of authority to predict involves a higher level of management dependence on the computer than authority to act. While the models on which the computer bases its action normally involve prediction, the potential impact of computer based prediction is often greater than computer action. Computer originated actions may adversely affect the firm's position at a point in time. However, actions relate to the operating sphere while predictions are the basis for planning. Thus, inaccurate prediction may have a damaging effect on the firm's activities for months or

years while erroneous actions can be corrected in days or weeks.

The chance for successful computer based prediction is ironically reduced by the very nature of management-computer interaction. Since predictions are often based on relatively sophisticated models, management is frequently hesitant to accept the computer's prognostication until they have gained experience with the system and had an opportunity to "see how it does". With the passage of time management's satisfaction with predictions which are verified by subsequent experience increases. However, as time passes the modeled environment may change -- the original models may become less and less applicable. Finally at that point when management is ready to take action based on the computer's predictions the models may be completely outdated and no longer accurately represent the decision environment. When this happens the stage is set for disillusionment or worse. It is considerations such as these which argue strongly for management involvement in the system design process, familiarity with system structure, and understanding of models on which system decisions and predictions are based.

Characteristics of Successful Systems

While specific functions performed by successful systems are as varied as the managements to which they contribute, four common characteristics of successful systems or perhaps more correctly the environment in which successful systems operate, can be noted.

1. The system is founded on management's conception of the decision environment.
2. The user-manager understands the system structure.
3. The system is based on disaggregated data files.
4. System development has proceeded to increasing levels of sophistication through a process of gradual evolution.

Management's Conception of the Environment

If a system is to provide meaningful information to a particular management it must reflect that management's priorities and provide information of a type and in a form which is assimilable in the context of existing management decision processes. In most situations this requirement specifies that the information must be selectively generated -- management is simply incapable of assimilating reams of paper -- and must be based on accepted measures -- output must relate directly to management conceptions of processes occurring in the monitored environment.

In order to meet this requirement an information system must be based on explicit models of the environment provided by management. In most instances management's initial system definition is stated in qualitative "business terms". Before meaningful specifications can be established this frequently vague and ambiguous initial statement must be refined and restated in explicit terms. Factors considered relevant in the decision environment must be defined and differentiated from those to be excluded.

Management Understanding

Management must be involved in this quantitative specification of system boundaries. They must understand and accept the conceptual structuring of system requirements in terms sufficiently explicit to define the measures and analytical procedures to be encompassed by the system. If this level of communication is not achieved, it may be impossible for those concerned with system formulation to develop a system which will be used.

But, it may be argued, this means involving management in wholly unacceptable detail. "Management is appropriately concerned with the big picture. It is unrealistic to expect them to become involved in questions of measurement." The response to this objection is to reiterate the area of eventual system application -- matters of company policy. It is difficult to

conceive of a point in the decision-making process at which involvement is more warranted than in insuring precise description of the decision structure -- unambiguous system specification.

The process of explication often uncovers the not altogether surprising fact that various members of management have different implicit conceptual models of the decision environment. Making these models explicit removes the ambiguities which permit vague words to mean different things to different people. Alternative representations are proposed and necessitate the creation and validation or rejection of more than one model.

As with any other specialized tool the information system must be carefully designed to meet the specific requirement of the craftsman who will use it and the user must understand its function and capabilities. There is no such thing as a generalized information system. It is difficult to conceive of a more specialized and highly segmented market than that for management information. The products which have gained acceptance in this market have been one of a kind special orders produced with careful attention to the needs and preferences of the ultimate user.

The Disaggregated Data File

At the heart of every successful information system is a disaggregated data file -- a file in which information is maintained in detailed time sequence as it is generated. As new inputs are received they are maintained along with existing data rather than replacing or being combined with existing information. New data are not combined with old to form sums, averages, or aggregate distributions. As a result, structural biasing through aggregation which destroys much information value is avoided.

Figure 3 illustrates the concept of a disaggregated data file based on data from invoice records. The disaggregated customer file contains the name, address, demographic, and financial experience records for particular consumers. Each transaction is recorded in chronological order in the file so that at any

point in time it is possible to recreate the company's interactions with each consumer over time. In a similar manner the product file is organized to reference a detailed chronological sales record.

The importance of a disaggregated file rests in part on the evolutionary process through which successful information systems develop. Although an information system may initially be designed to perform strict limited functions, as management gains experience these functions change. If data are initially structured (aggregated) to meet first stage requirements, later modification of system functions necessitate costly file reorganization.

The existence of a disaggregated file facilitates system evolution. Given access to detailed chronological data the manager is able to test new concepts and ideas against historical data asking the question "what would have happened if we had used these criteria in our monitor system -- performed this analysis to isolate actionable situations?". In the first stages of system development it is simply impossible to anticipate the direction of later advancement. Aggregate data files may preclude highly profitable system modification. The disaggregated data file provides the flexibility which is the prerequisite of intelligent system evolution.

Design for Evolution

Successful information systems are designed to permit expansion and change. As indicated above, the disaggregated data file is a key element in system flexibility. In addition, data files must be designed to permit expansion. Variable, rather than fixed record length file structures and self-expanding file constructs are basic to the well planned system.

As management gains experience in working with well organized and accessible data they become increasingly interested in and prepared to use more advanced analytical procedures. The system's analytical structure must not preclude this advancement. Programs must be organized to permit experimental use of new

techniques as well as the permanent incorporation of additional capabilities as part of the standard system configuration.

Characteristics of Managers Who Implement Successful Systems

In view of the key role which management plays in the specification and design of information systems, it is not surprising to find that the managers who are successful in developing and using management information and control systems share common attributes which distinguish them from the average administrator.

There is no industry bias in the population of successful information system users. Successful systems have been designed and implemented in consumer and industrial product and service companies as well as financial institutions. The members of this heterogeneous group may, however, be distinguished from their colleagues on the basis of the following characteristics which they share:

1. Belief that corporate growth is limited by their ability to assimilate and act on information.
2. Willingness to consider change.
3. Willingness to become personally involved in explication.
4. Willingness to test preconceptions.
5. Willingness to modify "time proven" procedures.

The managers involved in system development must be interested in making better decisions. They must be sufficiently confident and mature to consider conflicting approaches to a problem and to choose between alternatives on the basis of rational evaluation. They must be willing to give up implicit models with which they have worked for many years if, when made explicit and tested, these models are found lacking.

Over and above the personalities of individual managers, the environment within the company must support a free interchange of ideas. If managers are

unable to separate ideas from men -- if they evaluate the worth of a concept in terms of the title of the individual proposing it -- steering committee sessions will quickly give way to corporate infighting.

Obviously no company has a management which is perfectly objective and devoid of all traces of negative attributes. In final analysis it is a question of the ability of proponents of information system development to promote and implement this concept and management's willingness to engage in this type of activity recognizing the commitment and orientation required for success.

Examples Illustrating Basic Concepts

The remainder of this discussion will be devoted to examples illustrating system concepts associated with the management impact of information systems. We will begin with a relatively simple retrieval system developed for a small electronics company. Using this management environment as an example, the initial impact of a simple retrieval system will be examined. This will be followed by consideration of the impact of system conversion to on-line operation and the introduction of monitor, advisory, and decision making functions. The impact of real time information acquisition and micro-analytic simulation will also be considered.

A Basic Information Retrieval System

In June 1964 I presented a paper to this association describing a basic marketing information system developed for use by a small company introducing a new product to a difficult to define market.[1] I believe this system provides

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[1] "A Basic Marketing Information System -- A Case Study in the Economical Use of Computerized Management Information Systems", A Talk presented to the American Marketing Association Meeting in Dallas, Texas, June 16, 1964.

a good point of embarkation for our present discussion.

This system was designed to perform two relatively simple but useful functions:

1. Repetitive clerical operations.
2. Data organization.

It was a relatively simple information system first implemented in 1961 by a small company entering a new and undefined market with a product which was totally new in concept and function. Among the most pressing problems faced by the management of this company was to gain as much knowledge as possible about the markets they were entering. Unfortunately, with limited finances, they were not in a position to conduct extensive market research and were forced to promote and sell their product while learning about market response and composition.

Initial sales were generated through media advertising direct to various market segments. Advertising was designed to generate orders and requests for further information. Inquiries sent directly to the company or forwarded by distributors were answered with brochures describing the product line. Products were sold both direct and through distributor-retailer channels. Literature was supplied to retail outlets but since the company wished to encourage retailer forwarding of requests for additional information, it credited retailers with sales made to customers whose inquiries had been forwarded even though the customer might order directly from the company.

The basic justification for this system was its performance of clerical functions including order processing, mail preparation, invoicing and billing, financial record keeping, and warranty maintenance at a lower cost than clerical personnel. The management information and control functions of this system were, in a very real sense, bonuses gained from the system by designing it to keep track of and organize the information which it encountered while performing routine clerical operations.

The system exhibited the prerequisite of success outlined above. It was closely geared to management's concept of the new market environment; management was deeply involved in system design; a disaggregated customer file contained records relating to customers and potential customers with whom the company came in contact; and its structure was highly flexible -- as experience was gained in working with the system management was able to modify and expand itsfunction and scope.

This system falls at the midpoint on the information recency dimension introduced earlier. The average process lag between receipt of information at the company and inclusion in the system was from one to four days. It is important to recognize that this configuration did not involve measures of market activity other than those derived from media received at the company.

Evaluated in terms of level of information aggregation this system falls at the item level. It is based on a detailed disaggregated data file.

From a standpoint of analytic sophistication the system encompasses the first four levels illustrated in Figure 2. Although fundamentally a retrieval system programs used to develop outputs generated by the system perform aggregation, arithmetic, and logical analyses.

The level of authority delegated to this computer is minimal being limited to retrieval and error check review.

When beginning system development this management established two major priorities:

1. To establish a profitable operation.
2. To evaluate the effectiveness of alternative marketing programs and the potential associated with selected market segments.

These concerns were manifest in the measures specified for this system which emphasized profitability and market response. A few examples may help to illustrate some aspects of this system's impact on management thinking.

Figure 4 illustrates a channel analysis report developed by the sales

manager to provide a summary of product sales through particular outlets. The report illustrated covers the first six months of 1962. The company's concern with effectiveness evaluation is manifest in the use of trade margins to generate estimates of the dollar gross margin obtained from each outlet in addition to dollar and unit sales.

Since the system processes orders through all channels and, in addition, handles all warranty cards returned by purchasers the sales manager was able to implement a simple algorithm to estimate the movement of goods through each distributor. The resulting estimate of existing inventory in each outlet is indicative of the value of an integrated data file.

Management concern with market structure and composition is illustrated by the direct sales analysis report presented in Figure 5. This report provides a percentage breakdown of sales to consumers, secondary schools, colleges, business, government, and foreign purchasers during the month of May 1962 for four of the company's products.

More detailed information regarding the composition of the market was obtained through analysis of information gained from warranty cards returned to the company by purchasers of their products. As an example of the data obtained from this source, Figure 6 provides a sample warranty analysis for a single product based on warranties received during the first five months of 1962. This report summarizes the number of units sold to each of five age groups and indicates the percentage of product sales accounted for by each of these groups.

In examining Figure 6 it is important to remember that the information contained in this analysis was included in the consumer file as a descriptive characteristic of consumers who had purchased the product in question and returned their warranty card with relevant questions answered. Thus, at a later point in time, when the company was ready to market a new product believed to have substantial appeal to 15 to 18 year olds, the information which

had gone into this report was available in the data file as a basis for a selective mailing to those consumers who were within the appropriate age group.

Our examples thus far have focused on analyses of sales and consumer characteristics but have not taken advantage of the media response information generated in the course of normal processing of inquiries resulting from advertising.

Figure 7 illustrates one type of analysis relating to the performance of a variety of media employed experimentally during the fourth quarter of 1961. This report indicates the number of inquiries and sales generated by each advertisement and publicity release appearing in the indicated media during the specified period. Advertisements appearing later than December 5, 1961, are not included in the analysis. This report was made possible through coding of advertisements using keyed addresses, and tracing ultimate sales to original inquiries by means of the information retrieval system. The data file facilitated matching consumer names and addresses from orders and/or warranty cards to original names and addresses on information requests attributable to specific advertisements.

Figure 8 provides a summary of the advertisements contained in Figure 7 ordered by cost per order produced. I believe that the usefulness of this type of data presentation is self-evident. However, Figure 8 also illustrates a potential problem. Had management simply specified a listing of media by cost per order without including cost or performance data they might have concluded that media producing no orders were among the best performers. The problem illustrated is, of course, a simple result of the computer's handling of division by 0. However, it is indicative of a class of problems which arises when management accepts "answers" without understanding their basis in data.

An On-Line Management Information System

The term "on-line" relates to systems in which direct man-machine communication is made possible by the use of remote access consoles through which manage-

ment may interrogate or, given appropriate programs, interact with the computer. The point of origin device is generally a teletype or comparable typewriter based machine, however, in some more recent systems, video display consoles utilizing a television like display unit in lieu of paper output have been employed. The use of television displays of the type illustrated in Figure 9 facilitate the rapid presentation of extensive displays. In most instances the manager using a display unit of this type has the option of obtaining hard copy through ancellary printers.

The on-line system may directly parallel the off-line or batch process system with the remote access console serving the same function as the card reader or input tape unit in the batch system -- communicating to the computer the desired report format and source references. With theintroduction of direct access the problem of communication language must be considered. The language problem is obviated in batch processing since clerical personnel code requests for card input. It is, of course, possible to continue using the card code structure on-line. The manager requesting information must then type onto his console the same information punched in the batch process request card. Since card codes are designed to be economically communicated and directly read by the machine this coding procedure requires that the manager work in terms of numeric code structures. While not an impossible requirement, this type of communication can be very frustrating.

Simpler communication is frequently achieved through the use of control words or function keys. In this case the manager uses a function specifier such as CHART, DISPLAY, or CALCULATE to indicate the action to be taken by the machine or presses a "function" key which generates the desired process code. The function specifier is then followed, usually in relatively strict format, by a series of nouns and adjectives indicating the data source on which the desired report is to be based. The statement "DISPLAY BRAND X SALES, 6/1/61 - 12/31/61, DOLLARS, UNITS, MONTHLY" is representative of this type of request.

From the executive's point of view, the easiest form of communication is achieved when the machine can be programmed to accept "free form English language requests". The language is never totally free form in that the machine has a limited vocabulary -- it is assumed that the person communicating with it will limit himself to a pre-determined set of topics. In the absence of strict formatting there is a danger of ambiguity leading to misunderstandings between manager and computer. However, proper preparation of report formats insures that the manager is made aware of that which the computer thought he wanted. If the report content indicates that the computer is "confused" the manager restates his request using a different word order or more explicit statement. With experience, the manager learns to avoid ambiguous expressions. In the interest of efficiency he is apt to develop the habit of communicating with the machine in relatively strict format despite the "free form" capability.

The output illustrated in Figure 10 was obtained via an on-line console (in this case a teletype machine) interrogating the retrieval system described above modified by the addition of on-line interrogation capability. In this instance the manager has typed the request, "REPORT DISTRIBUTOR ACTIVITY FROM SEPTEMBER 1965 THROUGH OCTOBER 1965".

The computer has interpreted the word "ACTIVITY" to mean "sales and estimated profit". This definition is consistent with management's emphasis on providing profit computations whenever sales figures are given.

Figure 11 illustrates a second request and the corresponding computer response which demonstrates another aspect of system design. Although not evident to the reader unfamiliar with file content the interrogation requesting "company" information in fact provided data applicable to only one region of the firm's operations and a limited portion of the product line. Through the use of a password convention the system had previously established the identity of the inquiring individual. The information displayed in response to the word "COMPANY" was at the highest level of aggregation to which this individual

had access on the basis of pre-determined priorities. In this instance the figures relate to the regional level for those product lines which are the legitimate concern of the individual originating the inquiry. This type of security control insures that each individual using the system will have access only to data which is relevant to him. Through the use of aggregation structures which interpret limiting words such as "company" in context of a need-to-know hierarchy the system designer may insure that, once correctly identified by the computer, the president making a request for "value of company inventory" will receive actual balance sheet figures while the custodian making a similar inquiry will be given the value of janitorial supplies on stock.

A Monitor System

Without expanding the system's capability measured in terms of the first three dimensions of evaluation management may elect to grant additional authority to the computer by developing programs which enable it to monitor the content of all accessed files. In taking this step management is delegating to the computer authority for review and referral. The information system is used to implement a policy of management by exception with the computer directed to review all relevant data and refer to management only those situations which meet previously established criteria.

Installation of a monitor system introduces a new class of management problems. Policies must be formulated for computer implementation. Vague descriptions of "the sort of situation we're looking for" must be reduced to explicit definitions of that which will constitute an exception appropriate for referral. These specifications must indicate the data to be reviewed, frequency of review, and criteria of selection.

Figure 12 illustrates one type of monitor report obtained by adding monitor capabilities to the retrieval system considered earlier. This report indicates that the computer has encountered a situation in which a statistically significant, adverse sales trend has been established in one outlet of a particular distribu-

tion channel class while other outlets in that same class are showing a favorable trend.

The Advisor Function

If management is willing to go beyond the specification of criteria for referral and establish procedures to be followed when specific situations are encountered (e.g., write a letter, schedule a salesman call) the computer may be programmed to recommend an appropriate course of action. System expansion to encompass this function implies movement along two of the previously specified dimensions. Additional authority must be delegated to the computer system and, in most instances, more complex analytical programs must be developed and tested. The implications of alternative policies are frequently examined by using historical data to establish the recommendations which would have been generation had each policy been implemented during a past time period.

The Decision Function -- Direct Computer Action

Once management gains sufficient confidence in the quality of computer based recommendations they may extend the authority granted the computer one step further along the delegation dimension illustrated in Figure 2. The computer may be permitted to take the recommended action (e.g., write the letter or send the order) subject to intermittent review by supervisory personnel.

From a systems standpoint, differences between the advisory and decision making functions are small -- it is as easy to program the computer to write a letter as to generate a report indicating that a letter should be written. However, from management's point of view the relative impact of the computer is greatly increased once it is given the authority to act on behalf of the company. There is great solace in knowing that someone with whom we share human sensitivities is going to determine whether or not action should be taken. Management has an at-times justified fear of what an unattended computer might do if left to its own resources.

Successful implementation of decision systems is a result of management understanding of the procedures controlling the computer's action and careful specification of the precise level of authority granted the computer -- the specific situations in which it is permitted to take particular actions. In some situations management requires approval by supervisory personnel of computer based actions involving a commitment of resources above a certain level or the generation of communication beyond a specified point. In all instances management establishes orderly review procedures to be followed as part of a continuing assessment of computer based actions. Just as policies implemented by human subordinates are reviewed to determine their continuing applicability it is necessary to insure that models governing computer actions continue to be applicable to changing market conditions.

Learning Through Simulated Experience

Given access to a detailed disaggregated data file and a flexible program structure, management is able to test the effect of alternative analytical approaches under historical conditions -- to determine what would have happened if a particular decision procedure had been employed during the period of simulation.

In simulating past conditions, the computer at the point of decision, has no more information than it would have had were it making its decision at that moment in the then existing environment. Data following the point of decision are totally unavailable and the system must function on the basis of analyses of conditions existing at the decision time and historical records of conditions which prevailed prior to that time. As the computer moves through simulated time, information becomes available only when it enters the simulated time period when the information was generated. Through this process, years of hypothetical operating experience may be simulated in a few months of research. Following extensive testing of alternative criteria the decision procedures which yield the highest performance in terms of previously defined measures are implemented in the operating system.

Real Time Data Acquisition

The concept of real time process control is not new. It normally involves a situation in which some aspect of the production process is to be monitored to determine whether or not specified conditions are being met; evaluated to determine whether corrective action is required; and, when required, modified to establish the desired conditions. Figure 13 illustrates a typical man-based real time process control system. A meter is used to sense the thickness of the material coming from between the rollers. The man watches the meter. As the man notes a deviation away from the desired thickness he turns the wheel to adjust the roller positions, the meter returns to the desired position and the material is produced in the desired thickness. The basic elements of this system are:

1. A means of sensing the environment.
2. A conditional referral procedure.
3. A feed-back mechanism permitting the environment to be changed.

Computerization of the process described above is a relatively common phenomenon. The man at the wheel is replaced by a computer which senses the environment and automatically makes adjustment to maintain desired thickness. In most instances the man who previously turned the wheel is then upgraded from operator third class to operator first class (automatic systems specialist) and given responsibility for insuring that the computer's ON indicator remains lighted as illustrated in Figure 14.

The development of real time information systems has not been quite so simple. Although the basic concepts of real time process control are, at first blush, applicable to information system design, conditions motivating real time management information acquisition are not always clear.

At the present time managers are faced with hardware capabilities which greatly simplify real time access. Rapid data acquisition has improved the quality of airline reservation services, inventory control, and environmental surveillance systems. There is, however, a natural temptation aggravated by the pressures of computer salesmen to apply the real time capability in situations

where there is no need for real time data acquisition. Movement in the direction of real-time processing, is along the information recency dimension of Figure 1 in the direction of shorter time lags between event occurrence and system notification of the event. More rapid file updating is not necessarily harmful. However, the emphasis on recency created by the real time capability can cause management to give undue attention to recent events while ignoring more significant long term trends. It is ridiculous for a management to be concerned, for example, if sales in a particular region between 10 o'clock this morning and 2 o'clock this afternoon are off by 5% from sales during the same period last week.

Simulation-Based Information Systems

Systems discussed in the preceding sections have, for the most part, been based on relatively simple arithmetic or statistical models relating limited data from the market place to a single dependent performance measure. Since the planning and implementation of marketing programs involves the coordination of many types of management activity information systems have been designed to make use of micro-analytic behavioral simulations of the market environment. Two characteristics of the marketing decision structure strongly motivate the use of simulation based systems.

1. Controlling conditions in the market are a function of complex human behavior and responses.
2. Management must influence actions and responses in the market through persuasion since they are unable to exert direct control.

Management information systems based on micro-analytic market simulation generally focus on the processes through which management attempts to influence behavior in the market. The models on which such systems are based encompass detailed representations of retailer, distributor, salesmen, and consumer and industrial purchaser behavior as well as competitive interactions in the environment external to the firm.

The data files associated with these information systems encompass measures of the extent and nature of inputs to the market environment generated by the company and its competitors. The objectives in developing a simulation based information system are to achieve an artificial (model based) environment structured to accept inputs of the type monitored by the information system, and to generate outputs comparable to those obtained from the real world environment.

Figure 15 illustrates the structure of a simulation based information system. Inputs from the market environment are reviewed and formatted by a pre-processor system before being transferred to the master data file. The data file serves as the reference source for the information system and provides the historical data base for simulation model initialization.

Management has the ability to interrogate the data file directly and obtain responses following procedures comparable to those associated with the basic retrieval system discussed earlier. This set of interactions is noted by A in Figure 14.

Management's use of the simulation model as a basis for testing proposed programs is illustrated by the interaction set indicated by B . Proposed plans are inputted to the information system which establishes hypothetical conditions for runs of the simulation model. Results obtained in the simulated environment are transferred to the information system which formats them for presentation to management. Following this process management is able to evaluate the conditional results of proposed programs using the same procedures and equipment employed to assess the current state of the market through traditional interrogation.

Once a program has been finalized the proposed plan is established as a reference and simulated measures based on the plan are generated for use by the monitor sector of the information system. As the plans are implemented in the market environment actual measures of market performance are compared with

simulated measures indicating the expected results of planned implementation. Significant deviation from plans becomes the criteria for monitor referral to management as indicated by C in Figure 14. The information system may be used to evaluate the results of research activities as well as operating plans, as indicated by D .

Development of a micro-analytic simulation involves procedures comparable to those followed when developing less complex models. However, due to the structural detail the extent of management involvement required in order to achieve the pre-conditions of successful system implementation are greatly increased. Model development begins with management definition of system scope and the objectives to be achieved through system use. Initial specifications establish boundary definitions for the simulated environment. As the development process continues increasingly detailed descriptions of behavior within key sectors of the environment and interactions between sectors are established. Models are designed to facilitate simulated generation of measures referencing key backlogs, delays, and transfer points at which the rate of product, information, or value flow may be monitored. Key decision and response elements are identified and factors influencing these processes are delineated. Hypothesized relationships between inputs and observable behavior are formulated in terms of measurements which permit validation of the model against data from the real world. Once validated at the function level, decision and response formulations are combined in a simulation structure encompassing artificial populations exhibiting actions and responses covered by these formulations.

The behavior of population groups within each simulation sector is described by accumulating simulated individual behavior. Population behavior may be summarized in terms of the proportion of purchases allotted to each brand (brand shares), changes in population attitude distributions towards brands or any other measures encompassed by the real-world information system.

Once the simulation models have been validated to management's satisfaction the simulation structure may be used to produce outputs over time comparable to those received as input from the real world market environment. As an example, Figure 16 illustrates the weekly purchases of ten drugs by one hundred members of an artificial doctor population during the simulated year, 1961. These simulated market shares may be directly compared against prescription data received by the information system as inputs from the real world environment during the comparable period.

Summary

This paper has examined characteristics of successful management information systems in use by operating and policy managers concerned with the marketing function. It has focused on attributes and impact of representative systems and characteristics of the managers responsible for their implementation. Four dimensions of information systems evaluation have been suggested: (1) information recency, (2) level of information aggregation, (3) analytic sophistication, and (4) degree of computer authority. Representative systems performing retrieval, monitor, advisory, and decision functions have been examined in the context of this dimensional framework. Management implications of on-line, real-time, and micro-analytic simulation based system structures have also been discussed.

AEA:maf
9-9-66

Figure 1

BASES OF SYSTEM EVALUATION

DIMENSIONS 1 and 2

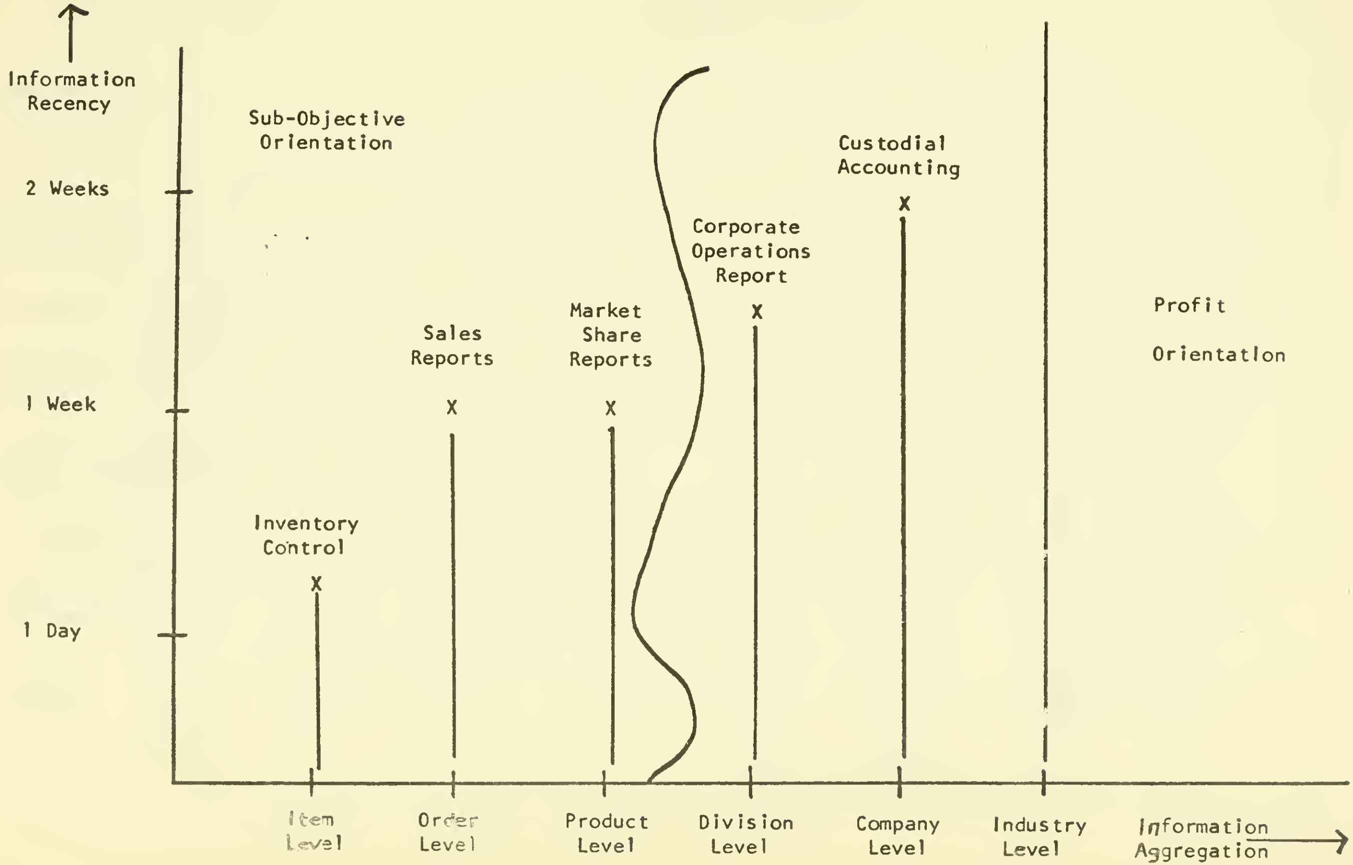


Figure 2

BASES OF SYSTEM EVALUATION

DIMENSIONS 3 and 4

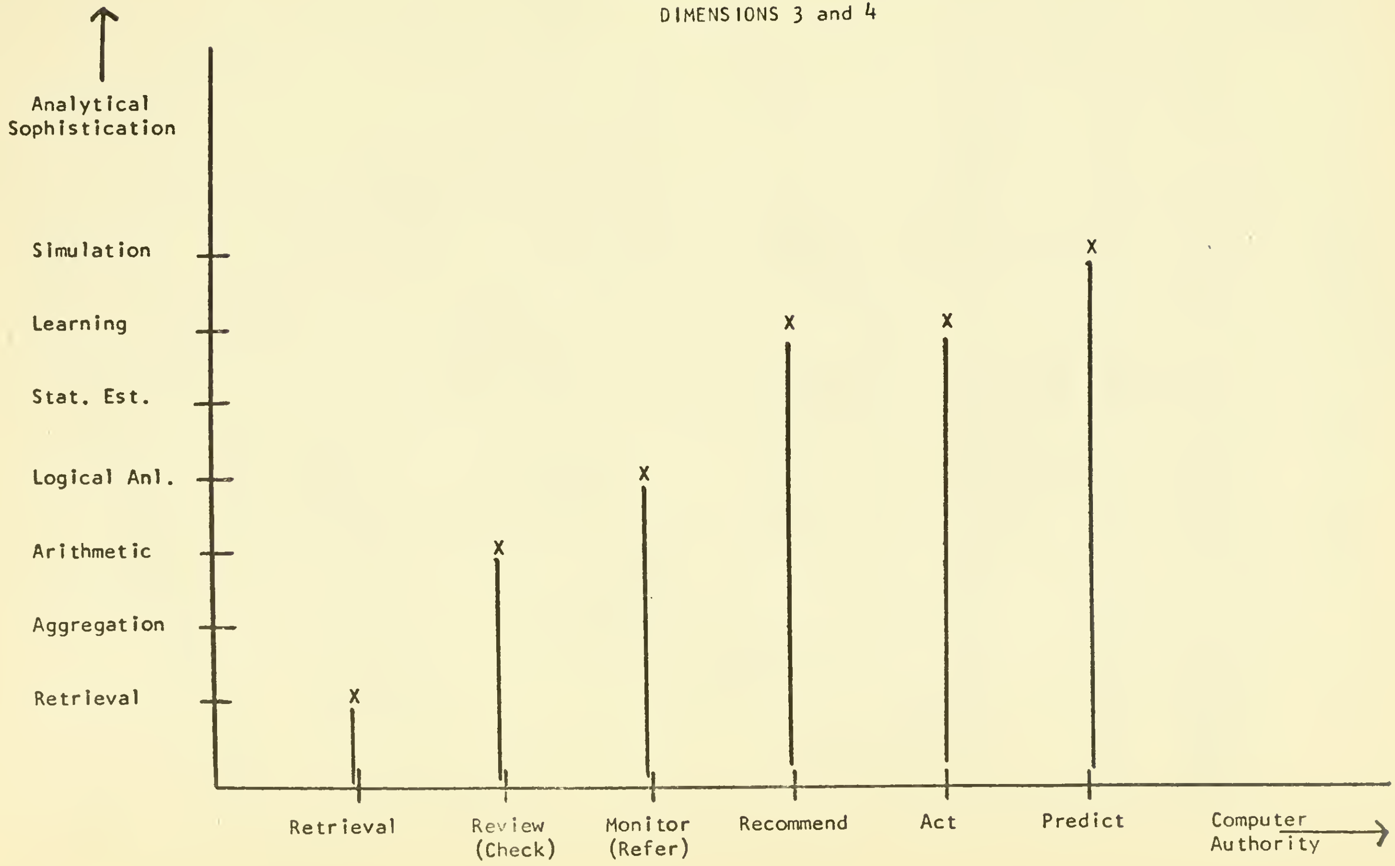
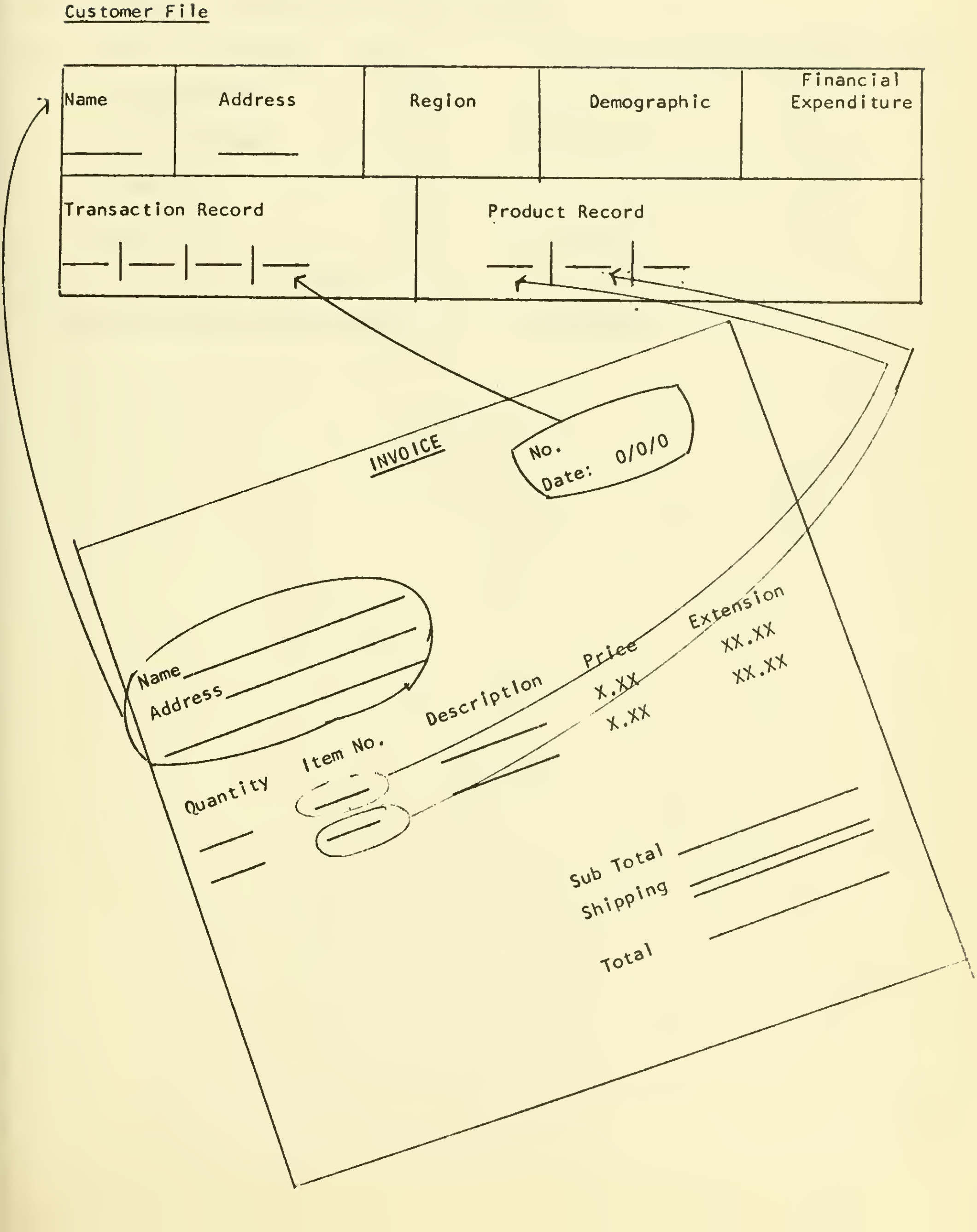


Figure 3

DISAGGREGATED DATA FILE



```
                         CHANNEL ANALYSIS REPØRT

CHANNEL CLASS -- ELECTRØNIC SUPPLY - [redacted]

DATES CØVERED - 01/01/62 THRØUGH 06/31/62

TØTAL SALES FØR PERIØD     UNITS          DØLLARS  EST. INVENTØRY UNITS

      -MIN[redacted]          158         9,756.50           32.

      -MIN[redacted]           26         2,619.50            8.

      -MAN[redacted]           12            72.00            0.0

      -C/R [redacted]          19           304.00           25.

TØTAL SALES TØ THIS CHANNEL              12,752.00

ESTIMATED CHANNEL GRØSS MARGIN            6,883.00
```

Figure 4

```
SALES DISTRIBUTIØN ANALYSIS -- DIRECT SALES -- PERCENTAGE BREAKDØWN

PERIØD CØVERED -- 05/01/62 THRØUGH 05/31/62

                     CØNSUMER  SECØNDARY CØLLEGES  BUSINESS  GØVERNMENT  FØREIGN

   -MIN[redacted]      48.0      18.0        8.0       9.0       7.0        10.0

   -MIN[redacted]      14.0      30.0       22.0      24.0       9.0         1.0

   -C/[redacted]       51.0       6.0       10.0      18.0       3.0        12.0

   -MAN[redacted]      12.0       0.0        9.0       3.0       3.0        73.0
```

Figure 5

```
WARRANTY ANALYSIS -- UNIT / PERCENTAGE BREAKDOWN

PERIOD COVERED -- 01/01/62 THROUGH 05/31/62

PRODUCT -- [redacted]

           AGE                  UNITS                 PERCENT

          15 - 18                129                   16.9

          19 - 25                 72                    9.4

          26 - 35                 78                   10.2

          36 - 45                246                   32.2

          46 -                   239                   31.2
```

Figure 6

```
                          CHRØNØLØGICAL MEDIA ANALYSIS


DATES CØVERED - 09/01/61 THRØUGH 01/01/62

REPØRT DATE -- 01/07/62

MEDIA CUT ØFF DATE -- 12/05/61

      MEDIUM             INQUIRIES  MEDIA CØST CØST PER  ØRDERS     CØST PER
                                               -INQUIRY             - SALE

BUSINESS WEEK-PR1           421.00       0.0       0.0   252.00        0.0
ELECTRØNICS ILL.            316.00    825.00      2.61    36.00       22.92
PØP. ELECTRØNICS-1         1167.00   1640.00      1.41    91.00       18.02
E.D.P. WEEKLY-PR1             4.00       0.0       0.0      0.0        0.0
PØPULAR SCIENCE-1          1874.00   3445.00      1.84   123.00       28.01
FØRTUNE - PR1                32.00       0.0       0.0    20.00        0.0
SCIENTIFIC AMERICAN-1       496.00   2310.00      4.66    77.00       30.00
ELECTRØNIC NEWS              92.00    330.00      3.59      0.0        0.0
ABSTRACT - PR1                5.00       0.0       0.0      5.0        0.0
SCIENCE WØRLD               396.00   1350.00      3.41    16.00       84.38
WALL ST. JØURNAL - 1        255.00    880.00      3.45    50.00       17.60
ELECTRØNIC WØRLD           1337.00   1495.00      1.12    55.00       27.18
PØP. ELECTRØNICS-2         1112.00   1640.00      1.47    48.00       34.17
NEW YØRK TIMES - 1         2057.00   3090.00      1.50   191.00       16.18
SCIENCE NEWS LETTER-1       310.00    430.00      1.39    93.00        4.62
PØPULAR SCIENCE - 2        2220.00   3445.00      1.55   104.00       33.13
SCIENTIFIC AMERICAN-2      1244.00   2310.00      1.86   213.00       10.85
SCIENCE NEWS LETTER-2       155.00    430.00      2.77    29.00       14.83
TECHNØLØGY REVIEW-PR1         6.00       0.0       0.0     4.00        0.0
AUTØMATIØN                  166.00    500.00      3.01    13.00       38.46
SATURDAY REVIEW              61.00    777.00     12.74      0.0        0.0
SCIENCE NEWS LETTER-3       126.00    430.00      3.41    13.00       33.08
PØP. ELECTRØNICS - 3       2873.00   1640.00       .57    97.00       16.91
SCIENCE NEWS LETTER-4        65.00    430.00      6.62    13.00       33.08
PØP. ELECTRØNICS - 4        758.00   1640.00      2.16    11.00      149.09
MATHEMATICS TEACHER          95.00    250.00      2.63     6.00       41.67
DATAMATIØN                  445.00    460.00      1.03     6.00       76.67
WALL ST. JØURNAL - 2         98.00    608.00      6.20    22.00       27.64
EL. DESIGN NEWS             289.00    350.00      1.21    11.00       31.82
NEW YØRK TIMES - 2         2940.00   3090.00      1.05   213.00       14.51
NEW YØRK TIMES - 3         2092.00   1989.00       .95   136.00       14.63
ELECTRØNICS PRØDUCTS         26.00    285.00     10.96     4.00       71.25
RADIØ ELECTRØNICS            36.00    340.00      9.44      0.0        0.0
INDUSTRIAL RESEARCH          90.00    410.00      4.56     9.00       45.56
FLEET ØWNER - PR1            91.00       0.0       0.0     2.00        0.0
MADISØN AVENUE - PR1          3.00       0.0       0.0      0.0        0.0
BUSINESS AUTØMATIØN          71.00    360.00      5.07      0.0        0.0
NEW YØRK TIMES - 4           25.00    164.00      6.56     2.00       82.00
WALL ST. JØURNAL - 3        125.00    385.00      3.08     4.00       96.25
SCIENCE NEWS LETTER-5        17.00     90.00      5.29     2.00       45.00
BUSINESS WEEK                93.00    835.00      8.98     4.00      208.75
SCIENCE                      17.00    230.00     13.53     2.00      115.00
```

Figure 7

```
                         MEDIA ANALYSIS--CØST PER SALE


DATES CØVERED - 09/01/61 THRØUGH 01/01/62

REPØRT DATE -- 01/07/62

MEDIA CUT ØFF DATE -- 12/05/61

      MEDIUM             INQUIRIES  MEDIA CØST CØST PER  ØRDERS      CØST PER
                                               -INQUIRY              - SALE

BUSINESS AUTØMATIØN         71.00     360.00      5.07     0.0          0.0
ELECTRØNIC NEWS             92.00     330.00      3.59     0.0          0.0
RADIØ ELECTRØNICS           36.00     340.00      9.44     0.0          0.0
SATURDAY REVIEW             61.00     777.00     12.74     0.0          0.0
SCIENCE NEWS LETTER-1      310.00     430.00      1.39    93.00         4.62
SCIENTIFIC AMERICAN-2     1244.00    2310.00      1.86   213.00        10.85
NEW YØRK TIMES - 2        2940.00    3090.00      1.05   213.00        14.51
NEW YØRK TIMES - 3        2092.00    1989.00       .95   136.00        14.63
SCIENCE NEWS LETTER-2      155.00     430.00      2.77    29.00        14.83
NEW YØRK TIMES - 1        2057.00    3090.00      1.50   191.00        16.18
PØP. ELECTRØNICS - 3      2873.00    1640.00       .57    97.00        16.91
WALL ST. JØURNAL - 1       255.00     880.00      3.45    50.00        17.60
PØP. ELECTRØNICS-1        1167.00    1640.00      1.41    91.00        18.02
ELECTRØNICS ILL.           316.00     825.00      2.61    36.00        22.92
ELECTRØNIC WØRLD          1337.00    1495.00      1.12    55.00        27.18
WALL ST. JØURNAL - 2        98.00     608.00      6.20    22.00        27.64
PØPULAR SCIENCE-1         1874.00    3445.00      1.84   123.00        28.01
SCIENTIFIC AMERICAN-1      496.00    2310.00      4.66    77.00        30.00
EL. DESIGN NEWS            289.00     350.00      1.21    11.00        31.82
SCIENCE NEWS LETTER-3      126.00     430.00      3.41    13.00        33.08
SCIENCE NEWS LETTER-4       65.00     430.00      6.62    13.00        33.08
PØPULAR SCIENCE - 2       2220.00    3445.00      1.55   104.00        33.13
PØP. ELECTRØNICS-2        1112.00    1640.00      1.47    48.00        34.17
AUTØMATIØN                 166.00     500.00      3.01    13.00        38.46
MATHEMATICS TEACHER         95.00     250.00      2.63     6.00        41.67
SCIENCE NEWS LETTER-5       17.00      90.00      5.29     2.00        45.00
INDUSTRIAL RESEARCH         90.00     410.00      4.56     9.00        45.56
ELECTRØNICS PRØDUCTS        26.00     285.00     10.96     4.00        71.25
DATAMATIØN                 445.00     460.00      1.03     6.00        76.67
NEW YØRK TIMES - 4          25.00     164.00      6.56     2.00        82.00
SCIENCE WØRLD              396.00    1350.00      3.41    16.00        84.38
WALL ST. JØURNAL - 3       125.00     385.00      3.08     4.00        96.25
SCIENCE                     17.00     230.00     13.53     2.00       115.00
PØP. ELECTRØNICS - 4       758.00    1640.00      2.16    11.00       149.09
BUSINESS WEEK               93.00     835.00      8.98     4.00       208.75
```

Figure 8

Figure 9

VIDEO DISPLAY UNITS

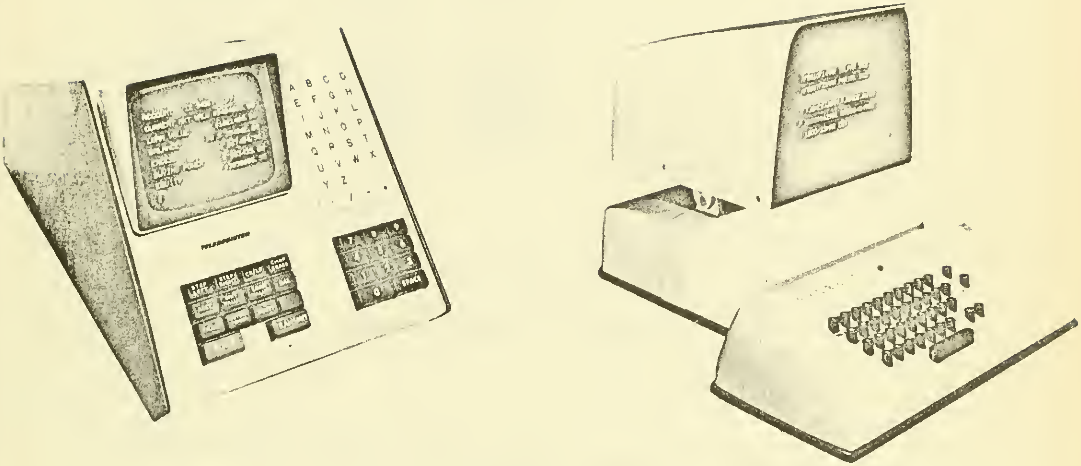
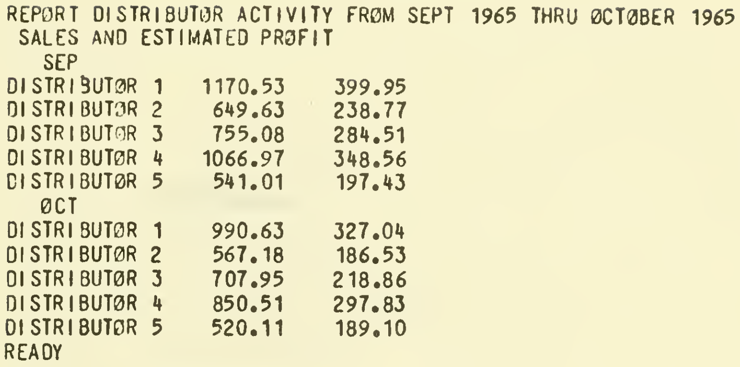


Figure 10

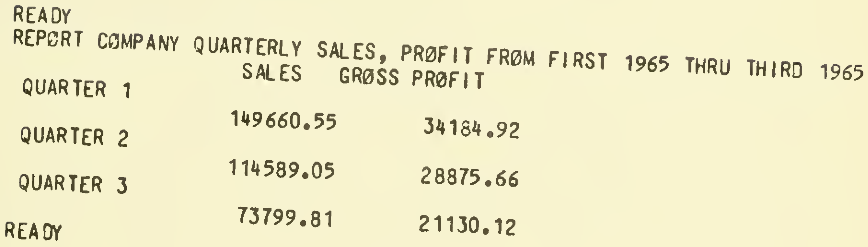
ON-LINE OUTPUT -- EXAMPLE 1



```
REPØRT DISTRIBUTØR ACTIVITY FRØM SEPT 1965 THRU ØCTØBER 1965
 SALES AND ESTIMATED PRØFIT
   SEP
DISTRIBUTØR 1    1170.53     399.95
DISTRIBUTØR 2     649.63     238.77
DISTRIBUTØR 3     755.08     284.51
DISTRIBUTØR 4    1066.97     348.56
DISTRIBUTØR 5     541.01     197.43
   ØCT
DISTRIBUTØR 1     990.63     327.04
DISTRIBUTØR 2     567.18     186.53
DISTRIBUTØR 3     707.95     218.86
DISTRIBUTØR 4     850.51     297.83
DISTRIBUTØR 5     520.11     189.10
READY
```

Figure 11

ON-LINE OUTPUT -- EXAMPLE 2



```
READY
REPØRT CØMPANY QUARTERLY SALES, PRØFIT FRØM FIRST 1965 THRU THIRD 1965
                      SALES    GRØSS PRØFIT
 QUARTER 1
                    149660.55        34184.92
 QUARTER 2
                    114589.05        28875.66
 QUARTER 3
                     73799.81        21130.12
READY
```

Figure 12

SAMPLE MONITOR OUTPUT

```
              MONITOR REPORT
PRODUCT - [redacted]
BASIS - DOLLAR SALES TREND
DATES COVERED - 01/01/64 - 0/08/64
CHANNEL - ELECTRONIC SUPPLY.
OUTLET -[redacted]
AVG. CHANNEL SALES     800.  PROFIT    250.  TREND     +46.
THIS OUTLET            210.             62.            -34.
```

Figure 13

REAL TIME PROCESS CONTROL

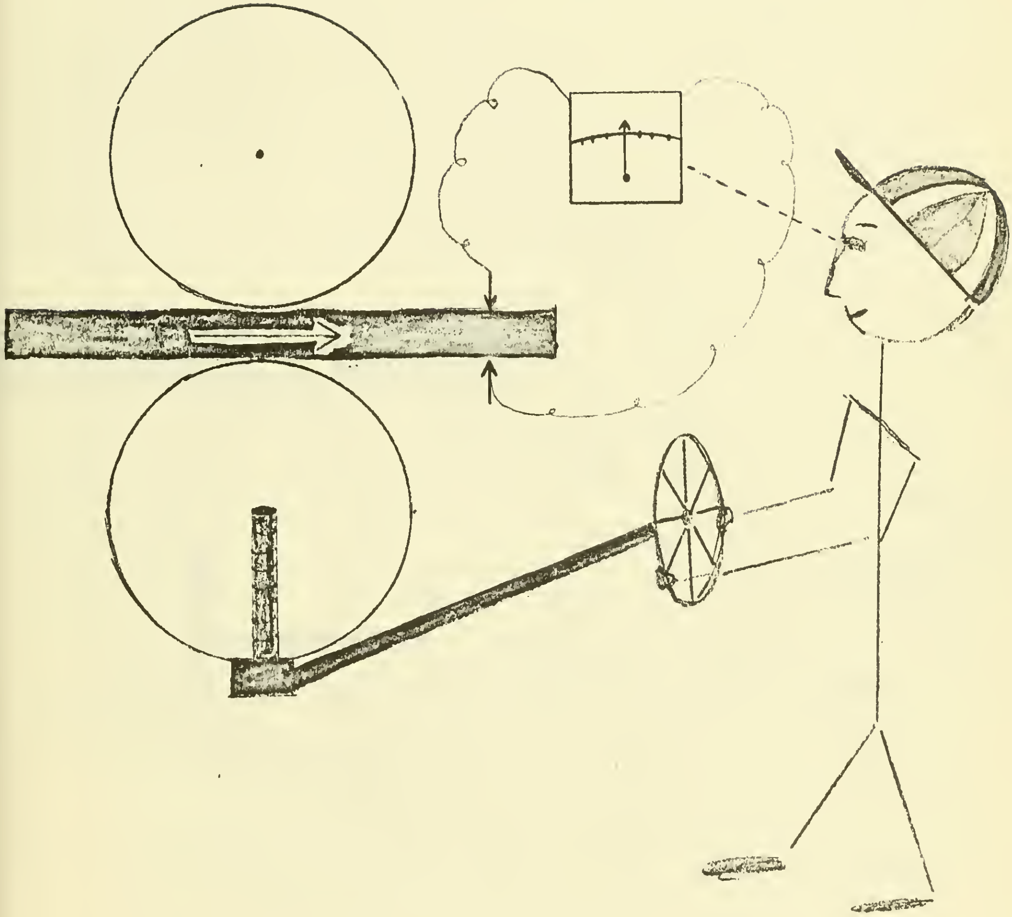
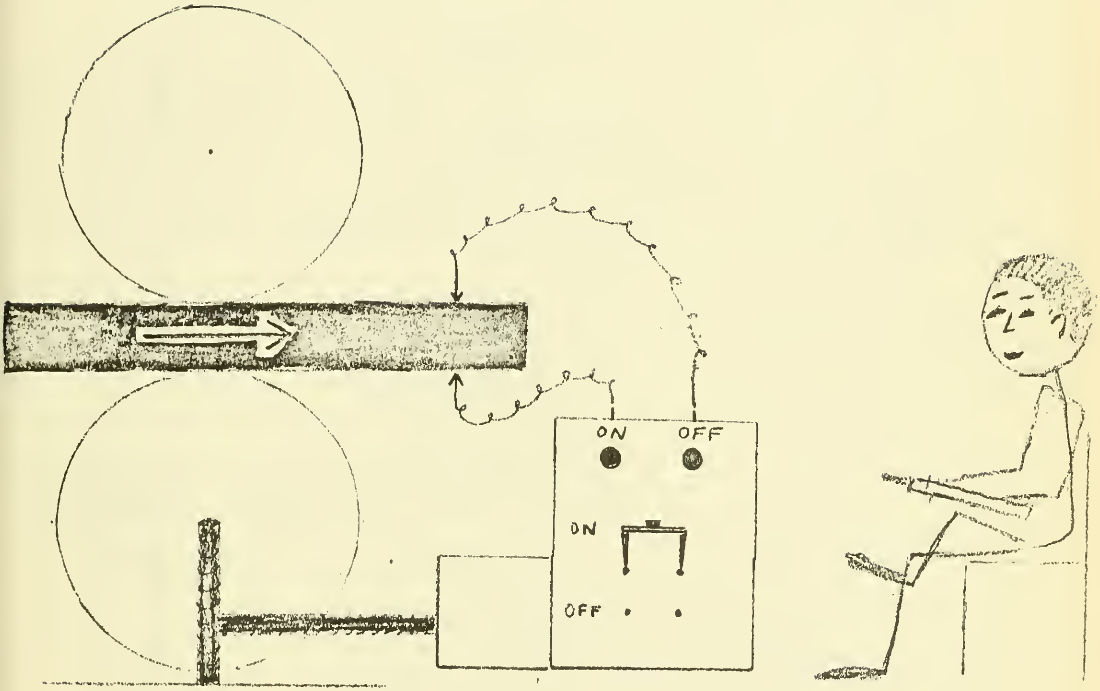


Figure 14

MACHINE AIDED REAL TIME PROCESS CONTROL



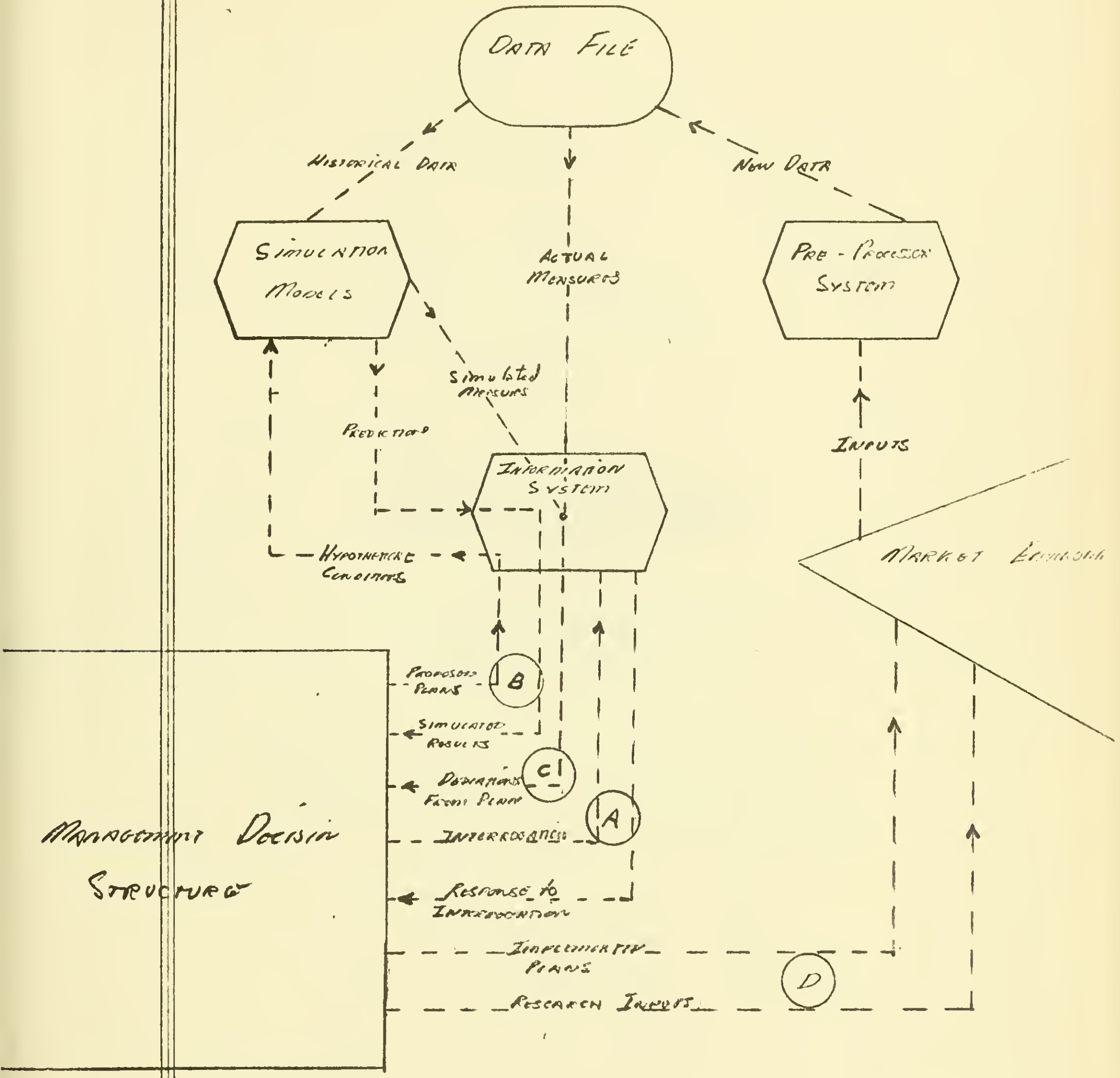


FIGURE 15 -- SIMULATION BASED SYSTEM STRUCTURE

SIMULATION TECHNIQUES IN THE ANALYSIS OF MARKETING STRATEGY

208-66

Arnold E. Amstutz
Henry J. Claycamp

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SIMULATION TECHNIQUES IN THE ANALYSIS OF MARKETING STRATEGY

by

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SIMULATION TECHNIQUES IN THE ANALYSIS OF MARKETING STRATEGY*

Arnold E. Amstutz
Henry J. Claycamp

INTRODUCTION

The complexity of the marketing process and the losses which frequently result from making poor decisions have caused marketing scholars and practitioners to constantly search for better ways to predict the outcomes of alternative strategies. The typical marketing manager has little direct contact and virtually no control over those whose actions ultimately determine the success or failure of his strategies. Changing competitive conditions and consumer circumstances; interactive effects of advertising, product quality, price and distribution; and time delays in response make it difficult to evaluate, much less predict the effects of a specific program.

In spite of these difficulties significant progress has been made recently in developing new methods of analyzing complex interactive systems and decision processes such as those found in marketing. In particular, the development of simulation techniques which utilize the amazing computational power and data handling capacity of large scale computers has greatly increased researchers' ability to

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*This working paper is based on material presented at the Application of the Sciences in Marketing Management Symposium at Purdue University on July 12, 1966.

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handle complex problems through the use of formal, symbolic models.

The purpose of this paper is to assess the current use and future potential of computer simulation in marketing. In the first part of the paper we will discuss the nature of the technique, some of the advantages and limitations associated with its use, and review selected studies which indicate how it has been applied to marketing problems. In the second part we will discuss the steps involved in developing and testing a complex simulation and describe the structure of a microanalytic model of buyer behavior which was designed to aid management in the generation and evaluation of alternative strategies.

CHARACTERISTICS AND APPLICATIONS OF COMPUTER SIMULATION

Although the term "simulation" has been used to describe many kinds of applications of abstract models to real situations, most operations researchers limit the meaning to:

1. the use of models which represent the essential elements of a real system or operation; and

2. the use of observations of the model's output or behavior under different experimental conditions to test hypotheses or make predictions about the real phenomenon.[1]

---

[1] For example see [5] and [10].

In computer simulation the model consists of mathematical and/or logical statements and the computer is used to calculate specific outcomes when different data inputs, parameter values, or structural relations are specified.

ADVANTAGES

The approach given above stands in sharp contrast to the classical use of symbolic models where general analytical solutions are used to deduce a model's properties. Since analytical solutions are not required in performing simulations, simple models are not essential for reasons of tractability. If necessary, hundreds of mathematical and logical statements and thousands of variables can be used to build a realistic model of a complex system. Freedom from the "solution constraint" allows the researcher to let his judgment about the importance of a variable be the primary determinant of whether or not it should be included in the model. It has also stimulated the development of new kinds of models which can be used in a wide variety of problem situations. Some of these models will be described later in this paper.

Morgenthaler (10) points out that simulation can be used to overcome other difficulties encountered in applying the scientific method of investigation. Once a simulation model has been found to be a reasonable representation of the phenomenon under study, observations of the behavior of the model can be used to derive and/or test

new theories and hypotheses. This use of a model is particularly important when it is difficult or impossible to induce experimental changes and observe responses in the real environment. The idea of "artificial experimentation" with a model to deduce the effects of possible changes in real-world structure is not unique to simulation. Indeed, such deductions are the ultimate goals of most model building efforts. The advantage of simulation is that it allows these experiments to be conducted in a more complex, and usually more realistic, "artificial environment" than would be possible using purely analytical methods.

However, it should be pointed out that the ability to handle large scale models through simulation does not imply that complexity is desirable for its own sake. The principle of parsimony has not been repealed by the advent of the computer. In many cases the power of the simulation method can best be exploited by creating complex systems made up of relatively simple components. Where this is possible one can combine the advantages of parsimonious representation of microstructures with the ability to handle highly complex macrostructures. The term "micro-analytic simulation" is used to describe this approach to model building.[2]

The characteristics pointed out above make simulation a valuable marketing tool. There is little difference between using a

---

[2] For an excellent example of this approach see [11].

simulation as an experimental device for evaluating theoretical models and using it to derive the implications of alternative marketing strategies. Thus, artificial experimentation through simulation can be used to produce direct inputs to management decision making.

More specifically, the major advantages of using simulation in marketing can be summed up as follows:

1. The simulation process can be used to explore the implications of management's perceptions about the external environment. Executives, salesmen, agency personnel, researchers, and other members of the firm's marketing team can pool their intuitive resources and readily available market data in order to come up with a series of statements about the behavior of consumers, middlemen, and competitors and the effect of specific marketing actions. Researchers can translate these statements into mathematical and/or logical analogues, and put them together in the form of a simulation. If the process is done well, the output of the simulation has some claim representing the implications of the perceptual inputs that went into the model. Aberrant results may be traced to errors in the interpretation of manager's statements or, more significantly, to errors or inconsistencies in the original formulations themselves.

2. Simulations can be used to integrate and systematize large quantities of information obtained from past marketing research studies and secondary sources. A model helps to place formerly isolated pieces of data in perspective and often increases the net amount of information that can be obtained from them.

3. A simulation model which has been accepted as a reasonable representation of the real world can be used to guide future research activities. Given confidence in the over-all structure of the simulation, the researcher can perform sensitivity analyses with respect to specific parameters or component characteristics. In addition to gaining general knowledge about the system's performance characteristics, these tests help to indicate what kinds of additional empirical or theoretical research will have the greatest impact upon the overall accuracy of the model. Research can then be concentrated in areas where potential results are known to be important.

4. Once a market simulation model has been validated to the satisfaction of both research and management personnel the method of artificial experimentation can be used to derive forecasts of sales levels, market penetration rates, profits, or other criterion variables conditional

> upon alternative specifications of the elements of the firm's marketing mix. The model can act as a kind of synthetic test market and can be used to screen alternative strategies without incurring the risk or expense of experimentation in the real world. While some experimentation in the real world may always be desirable to settle crucial policy questions and maintain a continuous check on the validity of the simulation, the ability to screen a larger number of test candidates through artificial experimentation is likely to produce both substantial savings and better candidates for experimentation in the real environment.

The payoffs outlined in paragraphs (1) through (4) above can be viewed as forming a kind of pyramid. One may start with a purely intuitive model, supplement it with data from existing sources and use the model to generate additional research leads which can in turn be fed back to improve the accuracy of the simulation. The apex of the pyramid is attained when both intuitive confidence in the model and formal validity checks on its performance are sufficiently positive to warrant its use as a tool for helping to solve "on-line" marketing decision problems.

Another possible application of the simulation model is in the area of managerial education. Military officers have participated in war games for many years. Modern war games simulate the outcomes

of strategic and tactical decisions by means of highly complex computer models. In the business game, executives or students who are responsible for managing companies in a simulated environment have the opportunity to obtain highly realistic experience in decision making and analysis. If the simulation model represents the marketing environment of a particular company or industry with reasonable accuracy -- that is, if it meets the validation criteria implied in connection with (3) and (4) above -- the training experience can be specialized to the point where executives from the firm or industry can take away specific knowledge about market characteristics that will be immediately useful in solving day to day problems.

Finally, the effort required to build and test a simulation model usually yields an important by-product in the form of the learning experience it affords managers and researchers involved in the study. A great deal of interaction is required between managers who have the intuitive grasp of the marketing process and researchers who have the technical skill to create the simulation. This is particularly true if the model is designed to be used as a direct aid to decision making. The manager is forced to be specific about his assumptions as to how the market operates -- more specific than he is used to being in his normal day-to-day activities. Moreover, he is asked to contribute to the resolution of "ify" questions about how various outputs of the model will be used in decision making.

On the other side, the researcher must always be alert to

questions of managerial relevance and is often required to compromise in matters of technical formulation and elegance. While the task of assimulating unfamiliar concepts and communicating in an unfamiliar vernacular can lead to friction between manager and researcher, experience has shown that open minded and intelligent attempts to resolve these problems will pay off. The effort devoted to specifying assumptions in precise terms (the computer is totally unforgiving of ambiguity), carefully considering and integrating all available data, and identifying important variables and interactions to be included in the model, invariably leads to new insights about the firm's marketing environment and policies even before simulation runs are ever performed.

PROBLEMS

While there are many advantages and payoffs associated with the use of a complex simulation there are also important problems and limitations.

First of all, computer simulation is inherently expensive. Skilled model builders, computer specialists and substantial amounts of computer time are required to develop, test, and maintain a large scale system. Even relatively simple models can involve major expenditures if special studies are required to generate data for parameter estimation and model validation. While the cost of a project is obviously related to its scope, some idea of the level of expenditure

often involved is indicated by a recent survey of simulations of social, political, and economic systems. Of 41 projects for which financial information was available, 20 were reported to have cost between $10,000 and $100,000 and 17 over $100,000. Some projects involved expenditures of as much as $2,500,000. [1]

The creation and use of complex simulation systems also involves significant technical problems. Foremost among these is the question of system design for efficient computation. Although nearly anyone with elementary knowledge of computer programming can translate individual elements of a model into computer language, considerable skill is required to design a large scale simulation. The basic problems involve program overlay and file manipulation. Simulation programs of the kind emphasized in this paper are frequently too large to fit into the core memory of a computer. Under these conditions it is necessary to divide the program into segments and bring each segment into core as needed to process information. Similarly, most simulations must bring large quantities of data into core, process it, and return it to external files. While many computers can perform calculations at the rate of a million or more per second, the operations required to transfer programs and data between internal and external memory take place at a much slower rate. Hence, it is essential to minimize such activity. Seemingly trivial differences in the organization of data files and the order in which computations are performed can make the difference between an efficient simulation system and one that is economically impractical to operate.

Care must also be taken in the design of systems which will permit experimentation with different data inputs and parameter values. In some cases special input editor routines which identify different classes of control information and check for syntactical and logical inconsistencies may be required to reduce the effort involved in introducing these changes and lessen the chances of executing a costly simulation run containing input errors.

Output systems must also be designed carefully. The following general considerations are relevant. First, output from sectors of the model, and sometimes even individual functions, must be available for performance testing. Secondly, procedures must be developed for translating results obtained from simulation runs into measures which can be validated against real world data and/or used by management in decision making. Finally, efficient assimulation of information requires adequately labeled and conveniently formatted output -- the latter often involves consideration of graphical as well as numerical modes of display. The development of effective output procedures is usually a non-trivial part of the design of the total simulation system.

Formidable problems are also present in the testing phase. Most simulation models, even those of moderate size, contain so many parameters that it is impossible to do sensitivity analyses on all possible combinations of values. Hence, the researcher must use judgment or a formal technique such as random sampling to select specific

values to be included in sensitivity tests -- and hope that feasible, but untested, values will not cause the model to behave in an erratic fashion.

Ascertaining the validity of a simulation model is also a complicated task. The researcher must decide which aspects of the real system are to be used as evaluative criteria and how close the model output must be before it can be said to be valid. Although the purpose the model was designed to serve provides guide-lines for selecting variables to be compared, there is no simple way to determine when correspondence between model output and real world data warrants the conclusion that the model is a valid representation of the underlying process.

Finally, it is important to re-emphasize the limitations inherent in the use of any model. Regardless of complexity and amount of detail, a model is by definition an abstract of reality. If important variables are omitted, and inaccurate assumptions and data are used, a model cannot be expected to produce highly accurate output. This fact seems to be easily forgotten, especially when the output is nicely formatted and consists of precise numbers calculated by a computer.

MARKETING APPLICATIONS

The diagram of a marketing system shown in Figure 1 provides us with a convenient way to categorize applications of computer simula-

tion in marketing. Each block in the diagram represents a population of individual behavioral units whose decisions control the flow of product, information, and money throughout the system. Although it is possible to simulate marketing systems using detailed models of the decision processes of individual consumers, middlemen and manufacturers, few attempts to do so have been made. Most marketing simulations have focused on specific kinds of decisions or variables and have treated other aspects of the total system at a high level of aggregation or ignored them entirely.

For example, simulation has been applied to the study of physical distribution systems by Shycon and Maffie [12] and Kuehn and Hamburger [9]. In the Shycon and Maffie study a model describing the location of factories and customers, and the transportation costs from feasible warehouse locations was created. Since the model also included functions describing inventory carrying costs and warehouse operation costs at various volume levels, it could be used to calculate the total distribution cost of servicing the firm's customers with a specific warehouse configuration. By specifying different warehouse locations the costs of alternative distribution strategies could be evaluated.

The Kuehn and Hamburger study was also designed to evaluate the economic implications of alternative warehouse locations. However, this simulation does more than evaluate the cost of a given system. It also selects the locations to be tried in the model from a feasible

set which has been specified by the user. Although the heuristics employed to select locations do not guarantee that the optimal set will be found, they do insure that each location added and retained in the solution results in lowered costs for the total system. Both of these simulations ignore marketing factors other than physical distribution and treat the patterns of demand by consumers as given.

One important aspect of information flow -- exposure of consumers to formal advertising media -- has been simulated by the Simulmatics Corporation [13]. The Media-Mix model was developed to estimate the reach and pattern of exposures of alternative media schedules. Census statistics for specific market areas were used to create artificial populations of consumers with representative distributions of important demographic variables. Data from audience and readership studies were used to develop equations which would yield the probability of a particular type of consumer being exposed to a given media vehicle at a given point in time. The model gives each of the 2,944 simulated consumers an opportunity to be exposed to each insertion in a media schedule which has been specified by the user. The stochastic outcomes are recorded and tabulated in reports which show, by consumer type, the reach and frequency of exposure resulting from the insertion schedule. Since the content and format of advertisements are not considered the model does not provide information about the impact of the messages on consumer orientations.

Simulation has also been used to study decision processes of

marketing executives and consumers. For example, Howard and Morgenroth simulated the way one executive in a large manufacturing firm set prices on the firm's products [7]. After extensive interviewing of the executive, a model was created which described the way in which information from several sources was used to determine a specific price. Validation tests were conducted by selecting at random thirty-one pricing decisions made by the original executive and 130 decisions made by executives who were not studied. In each of the 161 cases the model predicted the price accurately.

Similar studies have been made of retail ordering and pricing decisions. March, Cyert and Moore created models of the decision processes employed by executives in one department of a large department store [6]. The performance of the model in predicting markups, markdowns, and sale prices was particularly impressive -- 188 of 197 markups, 140 of 159 markdowns, and 56 of 58 sales prices were predicted to the penny.

Simulations such as the above are of immense value in understanding and improving the way with which specific decisions are made. Moreover they have indicated that it may be possible to let the computer make many of the repetitive decisions which take large amounts of executive time.

Although it is known that at least one company had a detailed simulation model of consumer decision processes and buyer behavior, there are few published studies in this area. As far as we

know, the model of doctor prescribing behavior to be discussed later in this paper is the most detailed simulation of this nature to be constructed to date.

A recent article by Kotler demonstrates the use of simulation in the study of interactions between major elements of the marketing system [8]. Models describing market and competitive responses were created and used to analyze the results of new product introduction strategies for two hypothetical companies. Thirteen different strategies were postulated for each of the duopolists. Tests were run on each of the 78 strategy combinations and comparisons were made of profits and market share over a five year period. Kotler used these results to draw generalizations about the market position, rate of return and amount of risk associated with different classes of strategies. While the model is generalized, i.e., it does not represent identifiable companies and products, it is easy to see how similar models of a specific nature can be developed.

The Balderston and Hoggatt simulation of the lumber industry is one of the few attempts to use microanalytic simulation to study interactions between manufacturers, wholesalers, and retailers [4]. While the primary purpose of the project was to study market structure and the way it is affected by different message costs and various methods of determining the parties to a transaction, the authors indicate that with relatively minor modifications the model could be adapted

to permit evaluation of alternative marketing strategies. Experiments performed on the model made it possible to draw generalizations about the dynamics of market segmentation and the effect of the experimental variables on profitability and concentration at each of the levels of distribution. In addition to being an excellent example of microanalytic simulation the documentation provided in Simulation of Market Processes makes this book a must for anyone interested in the technique.

The Total Market Environment Simulation (TOMES) is, we believe, one of the most detailed simulations of a total marketing system now in operation.[3] This simulation currently being used as a marketing game, contains artificial populations of consumers, retailers, distributors, and salesmen. In the consumer model one thousand individual decision-making units are simulated in each of nine census regions of the United States. Each consumer is described by socio-economic characteristics, probabilities of being exposed to specific media; awarenesses of brand names; and attitudes towards retailers, product characteristics, appeals, and specific brands of product. Awarenesses and attitudes about brands of product are formed by selectively perceiving the communication content of advertisements in print media, television, point of sale displays, and word-of-mouth messages generated by other simulated consumers. Individual consumers make explicit decisions to

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[3] The Total Market Environment Simulation was developed by A. E Amstutz, H. J. Claycamp, C. R. Sprague, and J. D. C. Little. It is currently being used in marketing courses at M.I.T. and Stanford University. For a more detailed discussion of the use of the model see [3].

shop, purchase, and generate word-of-mouth messages as a result of interactions between socio-economic variables and perceived brand images of specific products.

Individual retailers and wholesalers form profit expectations on the basis of communications received from manufacturers and observed consumer demand. If they decide to stock a specific brand they forecast their own sales, determine order quantities, and set their own prices.

Participants managing competing companies in this simulated environment can employ amazingly realistic marketing strategies. It is possible to vary product characteristics of a specific brand, place individual advertisements in specific regional or national media, employ individual salesmen and allocate their efforts to specific kinds of retailers, use direct or indirect distribution, etc.

Realistic research can also be done to provide information for decision making and evaluation of strategies. For example, simulated commercial services provide audits of retail sales and inventories and readership studies of specific advertisements. In addition, consumer research can be done by purchasing surveys of random samples of individual consumers in any of the nine regions.

While this simulation system was designed to teach marketing planning and decision making, it can be used as a research environment for evaluating alternative strategies and developing new analytical techniques. It also serves to illustrate the kind of richness which

can be embodied in a microanalytic simulation and the variety of uses which such a model can serve.

DEVELOPING AND TESTING COMPLEX SIMULATION SYSTEMS: AN EXAMPLE

The remainder of this paper is devoted to a discussion of the process of developing and testing complex simulation systems.[4] The procedure outlined here has been followed in developing models of markets for food products, appliances, services, securities, and electronic sub-systems, in addition to the prescription drug market example discussed in this paper.[5]

GENERAL STEPS IN THE PROCESS

Micro-analytic simulations designed for marketing management use focus on the processes through which management attempts to influence behavior in the external environment. The steps followed in creating, testing, and implementing a market oriented behavioral simulation may be summarized as follows:

1. A conceptual framework encompassing relevant attributes of the problem environment is delineated.

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[4] The work described in this paper was done in part at the Computation Center of the Massachusetts Institute of Technology and Project MAC.

[5] A discussion of system elements common to these various applications is also given in [2].

2. Elements of the environment considered important by management are established as the focus of a systematic study of market activity.
3. A theoretical structure encompasing relevant elements and processes is defined.
4. Relationships between elements and processes are expressed in quantitative and measurable terms.
5. Observed or assumed relationships are summarized in systems of equations compatible with a computer simulation structure.
6. Individual functions, system sub-segments, and the total simulation are related to data obtained from the "real world" environment.
7. Criteria of validation and performance measurement are established and model precision and accuracy are determined.

The first step in this process is to develop a broadbrush description of important factors in the marketing environment. This qualitative description serves as the basis for a non-mathematical but orderly structure which can be used to define key elements and processes.

The second step involves the design of gross macro flow models encompassing previously defined elements and processes. These preliminary descriptions of market activity define major interaction patterns and, as such, serve as cornerstones of models providing more complete and detailed descriptions of actions within relevant sub-sectors of the marketing system.

The third, fourth, and fifth steps encompass quantification of relevant behavior and development of detailed sector models. The models produced at this stage must provide sufficient scope and

refinement so that a representation of the total relevant environment may be synthesized in a single computerized simulation.

The final steps in this process involve practical implementation. The performance of the simulation must be validated against data obtained from the actual environment in which management is operating.

In the final analysis, the objective of simulation development is to produce a usable model -- a model on which operations can be performed in lieu of operation in the actual physical environment.

THE PROBLEM ENVIRONMENT

The system discussed in this paper was designed to provide the management of an international drug company with a new aid in the analysis, evaluation, and planning of marketing activities.[6] The specific objective of the project was the design of a complete simulation of the prescription drug market which would enable management to investigate in a synthetic environment the implications of alternative strategies and policies without making the commitment of resources required for comparable investigation in the real world.

In developing specifications for the system, management directed that it should facilitate: (1) evaluation of promotional (media) effectiveness; (2) evaluation of salesmen (detail men) effectiveness; (3) testing of alternative policies and strategies for

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[6] Work on this simulation was started by the authors in 1963. Some aspects of the total system are still in development.

marketing particular products to given market segment; (4) evaluation of the probable success of new products at an early stage of market development; (5) assessment of the validity of management's understanding of the dynamics of the prescription drug marketing system.

While the constraints of this paper do not permit a complete exposition of the methodology or results of this activity, highlights of the development procedure and system performance are discussed in the following pages.

SYSTEM DEVELOPMENT IN DETAIL

In developing a simulation of competitive market behavior the firm and its competitors are viewed as input generators. The external market simulation is then designed to duplicate the response characteristics of comparable real world markets to the inputs generated by the competing firms.

Boundary Definitions

System development activity normally begins with a definition of the boundary conditions which limit the scope of the system to be developed. In most instances this preliminary specification is relatively crude. Management generally attempts to describe a limited number of sectors. The description may be of the form illustrated in Figure 2. This figure shows management's preliminary conception of the prescription drug market environment. Lines connecting various

sectors of the illustration indicate management interest in interactions between these market elements.

At the outset, management must also specify the objectives which they hope to achieve through use of the system once it has been developed, validated, and implemented. Objectives of the type defined above frequently determine whether a particular aspect of the environment will be included or excluded. Proposed applications also determine the level of detail and accuracy which management requires of the operating system.

Once the desired scope and objectives have been specified, macro descriptions of behavior within the environment to be simulated can be undertaken.

Macro Specification Development

Macro specification is designed to achieve two interrelated objectives. First, it initiates the process of quantitative model formulation. Second, it provides an opportunity for managers and researchers to establish the conceptual framework and preliminary definitions of key variables.

During the macro specification phase, major emphasis is placed on stating that which management knows, assumes, and hopes. Underlying assumptions about the nature of the environment are given close scrutiny. Boundary conditions established in preliminary discussions are refined to the point where the scope and detail of future analysis and evaluation may be established. Thus, macro specifications

formalize the preliminary model structure and establish the frame of reference for all subsequent model development.

Figure 3 illustrates this step in the process of system specification. Concepts illustrated in Figure 2 have been expanded through recognition of additional sectors and more complete definition of interactions between sectors. Flows of information, orders, prescriptions, and product have been identified.

Beginning with the company in the upper left hand corner of the flow chart, product flow is followed through wholesalers, chain outlets, pharmacies, and hospitals. Parallel order flows are noted from the wholesaler, hospital, and pharmacy levels. Distribution facilitating information generated by the company is indicated as an important input to salesmen, wholesalers, pharmacies, and doctors. Information inputs to the company include observer reports, salesman reports, panel research, and direct mail research.

Salesmen are represented as receiving information from the company and transmitting it to wholesalers, pharmacies, hospitals, and doctors.

Wholesalers are perceived as receiving information directly from the company and through its salesmen, transmitting orders to the company, and receiving product from it. The small oval to the left of the wholesaler sector indicates product inventory at the wholesaler level. Inputs to the wholesaler are indicated as originating in the hospital, pharmacy, and salesman sectors. Wholesaler salesmen are represented as order takers.

Pharmacies receive information from the company via its salesmen and some wholesaler salesmen. The possibility of both direct and through-salesman order procedures is noted. Inventory maintenance is indicated by the oval to the left of the pharmacy sector.

At the level of detail represented in the Figure 3 flow chart, the hospital is analogous to a pharmacy.

The doctor is described as receiving information from the company salesmen, media, and direct mail promotion. An additional source of information is represented by the information line leaving the lower right hand corner of the doctor rectangle and returning to that same sector. This line represents doctor interaction and the generation of "word-of-mouth" communication. The company receives information inputs about the doctor sector through observer reports, panel, and direct mail research.

The patient is shown as interacting with the doctor, receiving prescriptions under control of the doctor, and under certain circumstances, initiating refill procedures.

While competitors are not illustrated in this flow chart, the actions of relevant companies are considered in detail in the actual simulation.

The process of macro specification is frequently iterative. Initial specifications provide the basis for preliminary definitions which are then modified in the light of additional conceptual development, market studies, and data constraints. For example, once preliminary formulations for the drug market simulation had been developed,

substantial time was spent in discussing these formulations with members of management as well as practicing physicians. These interactions, as well as additional analysis, and empirical research led to refinement of the initial structures.

Data Requirements

Macro specifications refine boundary conditions to the point where specific data requirements may be established. Figure 4 summarizes representative data requirements associated with the drug market simulation. The two sections of this figure distinguish between data required for model structuring and initialization and that used as input during operating runs.

Data sources included monthly audits of drug store invoices, weekly audits of prescriptions written, audits of the distribution and content of journal advertising, quarterly reports from panels of doctors who recorded individual patient treatment, direct mail promotions, and salesman details (sales messages for specific drugs). Specialized research studies were also employed to determine doctor knowledge, experience, attitudes, and treatment procedures.[7]

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[7] Generation of the data required for a simulation of this magnitude is in many cases a monumental task. However, the pahrmaceutical industry is unusually rich in commercial data services, and the company sponsoring this simulation has an especially effective market research department. Hence, much of the data required for this simulation was already in existence and only needed to be integrated into the model.

Micro Specification Development

Once key decision and response elements have been identified the focus of model development shifts to micro specification. The first activity in this phase is the creation of detailed models based on management hypotheses regarding the problem environment and verified where possible by reference to behavioral theory and existing data. Working within the structure supplied by the macro specifications each decision point is described in terms of inputs to and outputs from that decision. Hypothesized relationships between inputs and observable behavior are formulated in terms of measurements which permit validation of the model against data from the real world. Each functional relationship is explicitly described in mathematical or logical expressions, and instructions for computer system design and programming are established.

Simulations of the type being considered here involve unusually complex computer programs. As a result, a major portion of micro specification normally focuses on the creation and testing of computer programs required for data packing, multi-level system control, and overlapped processing.

Description of a Decision Process

The conceptual framework summarized in the macro specification hypothesizes a doctor's decision to prescribe particular drugs for a patient exhibiting specified indications (illnesses). Treatment may take place in the doctor's office, a hospital, or the patient's

home. In any case, the fundamental problem facing the physician is selection of the appropriate therapy on the basis of his present knowledge, attitudes and experience.

Basic characteristics of the simulated drug selection procedure are summarized in the Figure 5 flow chart. This representation begins with the initialization of a doctor decision matrix, DRDECM. All elements of this matrix are initialized at zero. Thus, in the beginning the process the doctor is assumed to have no predisposition other than that reflected in his memory of associations and experience.

The indication(s) exhibited by the patient are noted by the doctor who recalls one or more drugs which might be used in treatment. This process is simulated by placing a 1 in the drug decision matrix position representing each drug which has been associated with treatment of the exhibited indication(s). If, upon completion of this process, the matrix is zero, the association process has failed and the doctor adopts a more complex procedure.

The second procedure involves a systematic evaluation of the doctor's past experience with relevant drugs. In the simulation, this process is initiated by setting appropriate drug matrix positions to 1 indicating that all drugs in the set are equally appropriate for consideration. The doctor deletes from consideration those drugs toward which he has a negative attitude as a result of previous experience. (Drugs previously used in treating each patient are recorded in the patient's treatment record to facilitate identification.) At this point in the decision process if the doctor is in a hospital all drugs

not included in the hospital formulary are deleted from consideration.

If the matrix is zero following this process, reconsideration is effected using modified standards as specified at location 10 in the flow chart. If the matrix contains one or more non-zero entries, the doctor's attitude toward each remaining drug is established. All drugs for which the resulting attitude is negative are then removed from consideration. If there are no drugs toward which the doctor has a positive orientation, no drug is used. If, on the other hand, one or more positive attitude drugs remain, the process continues to the entry specified as position 20 in the flow chart.

At position 20 the doctor considers each remaining drug in terms of his past success in achieving desired therapy in similar circumstances. Historically unsatisfactory drugs are rejected with the corresponding entries in the drug matrix set to zero. If, following this procedure, all drugs are eliminated, drugs deleted due to unsatisfactory historical performance may be reconsidered with a less stringent performance requirement.

If several drugs remain to be considered after either the initial or revised test, the doctor makes a choice based on his attitude toward each drug remaining under consideration. If his attitude toward all drugs is equivalent he will have an equal probability of choosing any one of the drugs. If, on the other hand, his attitude toward one drug is more favorable, his choice is biased in favor of that drug.

Additional Function Formulation

In a similar manner, each decision and response function encompassed by macro specifications is investigated. In some instances initial theoretical constructs are validated. In others, empirical evidence suggesting alternative constructs is obtained and the process of formulation is repeated for revised structures.

The final structure established by micro specifications includes processes through which the doctor is exposed and responds to media, conventions, salesmen, and word-of-mouth communication; evaluates indications exhibited by a patient; establishes desired actions, efficacy, and safety; and schedules the patient for a return visit.

Explicit Decision Representation

Decision and response functions are formulated and tested as probabilities since data from the real world environment are in the form of frequency distributions. Generation of explicit decision outputs for each cell within a simulated population requires conversion of the probabilistic statement into explicit yes/no decisions. A number drawn randomly from a rectangular distribution of range 0 to 1.0 is compared with the stated probability to determine each probabilistic event.

AN EXAMPLE OF THE SIMULATION PROCESS

Behavior within each sector of the simulated environment must be described in terms of interrelated decision and response functions of the type illustrated above. In addition to describing

functional relationships, the system designer must develop a representation of the dynamic processes which produce behavior observed within and between sectors of the market.

The operation of a micro-analytic simulation can be most easily described by discussing the basic flow of information through the system and examining characteristics of the processes through which behavior is created. The basic structure of the prescription drug market simulation is illustrated in Figure 6.

System Initialization

On entry, the system prints certain title and control information on output tapes (A-3) and the on-line printer. Tables developed by a pre-processor program are then read into core storage to establish information to be referenced during the simulation cycle and initialize operating parameters.

Doctor File Input

Each simulated doctor is described by the content of a doctor file record. Doctor files are recorded on tape sequentially by geographic region. A single doctor file is held in core at a given point in time. After simulating the doctor's activity for a specified number of weeks the file is updated to reflect his experiences, and written on tape. A new doctor is then read into core, and the procedure is repeated.

The Time Loop

As indicated above, the system is structured so that time is moved past each doctor in turn. This organization of the system is necessitated by the large size of the doctor file record which makes it impractical to move doctors in and out of core or to maintain more than one doctor in core at a given point in time.

During most simulation runs, the time period considered is one simulated year. The time step is one week and the time index (IT) proceeds sequentially from 1 through 52. Events occurring during a particular week are identified by a monotonic date code, which, during processing of the simulation, is referenced to the time index (IT).

Doctor Response to Media Promotion

During each week in simulated time, the publication frequency of each relevant journal is tested to determine whether it is published during the week under consideration. If a particular journal appears, the probability of the doctor then under consideration being exposed to that journal is developed. If, on the basis of this probability it is determined that the doctor will be exposed to the journal, each advertisement appearing in an advertisement schedule table for that journal is examined to determine whether or not the doctor will be exposed to, and assimilate any new information. When an advertisement is assimilated the doctor's response to the message is established and his memory updated to take account of information content. This process is continued for all media,messages and doctors at each point in time.

Direct Mail Response

The handling of direct mail response is structured in a manner analogous to media promotion. During each simulated week, a comparison is made to determine whether any direct mail pieces appear. If a direct mail piece is being sent during the week in question, exposure probabilities are developed to determine whether or not the particular doctor then being considered will be exposed to the specified mailing. If exposure occurs, assimilation probabilities are generated and, if on the basis of these probabilities it is determined that the doctor will assimilate portions of the communication, his response is determined and his memory updated.

Response to Salesman Detail

In developing a representation of the doctor's response to salesman communication, the probability of exposure is first determined on the basis of parameter values in the doctor file record which establish the probability that the doctor will receive a call from a salesman representing any one of the relevant companies. If the doctor is exposed to a salesman from a particular company the schedule of details (sales messages for a specific drug) presented by that salesman is examined to determine which details are being presented to doctors of the indicated specialty during the week under consideration. If a particular detail is presented and assimilation occurs, the doctor memory is updated. As in the case of all other communication response loops, this procedure continues until all sales messages have been considered.

Response to Convention Activity

Exposure to presentations at a convention is based on a convention schedule which specifies the probability of a doctor of a particular specialty and residence attending a convention held at a particular time. In keeping with the previously established procedure, the convention schedule is examined once each simulated week to determine whether or not a convention is being held. If a convention is being held, the probability of the doctor then in core attending that convention is determined and, if the doctor is found to attend the convention, procedures similar to those outlined above are used to determine exposure to and assimilation of relevant information.

Response to Word-of-Mouth Communication

Within the structure of the simulation, messages generated by doctors in a particular region are accumulated along with descriptors of the generating doctor in a table of word-of-mouth messages. Thus, when a particular doctor is in core, messages generated at various points in time by doctors preceding him are available in the word-of-mouth table. This table is referenced in a manner analogous to the schedule and content table discussed for other media. The probability of interaction between the doctor in core and the message generating doctor who preceded him is established. If the doctor is exposed to the word-of-mouth communication the probability of assimilation is developed in a manner analogous to other communication functions and the doctor's memory is updated to reflect the word-of-mouth interaction.

Treatment of Patients

The simulated doctor is exposed to patients from an artificial patient population which is supplied as an input to the simulation. An average patient load parameter in each doctor file record determines how many patients will be treated in a given week. In treating a patient the simulated doctor determines what drug or drugs, if any, will be prescribed for the exhibited indication(s) of the patient.

Once treatment has been decided upon, the probability that it will achieve desired results is established on the basis of clinical data. If it is determined that the treatment undertaken will not prove effective within a specified period of time, the patient is maintained in a backlog of patients who will return to the doctor at some time in the future. If the outcome of treatment is successful, the patient is for all practical purposes dropped from the model. In either instance, the trial and outcome (including possible side effects) of a particular treatment is noted.

After the first simulated week the doctor has two sources of patients: (1) patients in the population from which his original patient group was drawn; and (2) patients who require continuing treatment. During subsequent time periods the doctor's first source of patients is the returning patient file. After all patients previously treated and scheduled to return have been treated the doctor considers new patients from the outside population.

Generation of Word-of-Mouth Communication

As the doctor considers various drugs in context of the treatment during the simulated week, a record of his attitude toward his experiences is maintained. Following completion of the treatment cycle for a particular simulated week, this record is examined to determine whether the doctor will generate word-of-mouth communication regarding some aspect of his recent treatment experience. If such word-of-mouth communication is generated, communication content is established, dated, and stored in the word-of-mouth communication file for later referencing by other doctors.

Forgetting

At certain prescribed time intervals, the doctor's memory is examined to determine whether forgetting would have occurred during the lapsed time period. The memory record for each drug is examined and if forgetting has occurred, the record is reduced.

Time Cycle Combination

The basic process described above is repeated for each week in the simulated year for each doctor in the artificial population. Once the final week (IT=52) for a given doctor is completed, an activity report is generated and the doctor file record is updated to reflect his experiences during the simulated year. This record is then written on tape to serve as an input for simulation of future time periods.

Following completion of a given doctor record, the simulation returns to the point in the Figure 6 flow chart labeled "A", reads another doctor record from the tape file, and repeats the process as described. After all doctors have been considered for the specified period of simulated time, a final summary report is written and the simulation terminates.

TESTING

Once a simulation has been developed to the point where it can be used to produce artificial behavior, the emphasis shifts to testing. Although the ultimate test of any model is its usefulness, the stability, reliability and validity of a simulation should be ascertained before it is used as an operational tool.

Stability Testing

Stability tests are concerned with the reasonableness of the model's performance when it is subjected to different, but feasible, parameter values and input data, and run for substantial periods of time. The major problems encountered in stability testing are selection of apecific parameter values, definition of "reasonable" performance, and determination of an appropriate time period for the test.

Reliability Testing

Tests of reliability focus primarily on the question of reproducability of results. The basic problem is one of determining

stochastic variations of important outputs when different series of random numbers are used to determine specific outcomes within the system. Confidence intervals for important outputs can be established using various statistical techniques.

Validity Testing

Tests of validity are concerned with "truth." While reliability may be assessed using standard statistical techniques there are no objective measures of truth. Consequently, the researcher must turn to a subjective evaluation of the accuracy of the assumptions used to create the model and the consistency of its performance with theory and empirical data. In the final analysis, a model is realistic if it duplicates the relevant characteristics of the real phenomenon. For example, Turing has suggested that a model may be called "realistic" if a person knowledgeable in the subject being modeled -- i.e., a person having experience with the relevant reality -- cannot distinguish model output from output generated by the real system. Thus, once the validity of assumptions has been established tests must be made of model output.

The procedure normally followed in testing the validity of a micro analytic simulation is to proceed sequentially through analyses of individual functions, individual cell behavior and total population behavior.

Function Validation

Since the number of functions involved in a large scale simulation usually precludes exhaustive testing, this activity is usually limited to investigation of the performance of functions which are known to be essential to system operation.

Cell Level Validation

The objective of validation at the cell level is to establish that the behavior of an individual within the simulated population cannot be differentiated from that of a similar member of the real world population. Figures 7 and 8 illustrate the kind of output which can be obtained from the drug simulation for purposes of cell validation. Figure 7 summarizes the characteristics of a single ficticious doctor at the time when he began a particular week of simulated activity. The doctor in question is a general practitioner between 45 and 60 years of age in private practice in the midwest. He employs a nurse-receptionist and treats an average of 136 patients per week. Data following these descriptors indicate the doctor's media habits expressed in terms of circulation and exposure probabilities for major journals encompassed by the simulation. Company exposure probabilities indicate this doctor's historical frequency of interaction with salesmen from companies included in the simulated environment. The doctor's previous treatment experience is summarized at the bottom of Figure 7 in statistics showing seasonalized proportions of his practice devoted to treatment of relevant indications.

Figure 8 illustrates a simulated doctor's treatment of a geriatric patient suffering from an upper respiratory infection. The output indicates that on a prior contact with this patient the doctor prescribed two drugs. On this visit the patient's condition is improving; however, the doctor is now concerned with an anxiety reaction and undue fatigue exhibited by the patient. Noting this condition, the doctor is seeking anti-depressant and general tonic actions. The Figure 8 output provides a detailed account of the process through which the doctor elects to prescribe a new drug.

The Turing test of cell behavior was conducted by developing exhibits, such as shown in Figures 7 and 8, which describe patient treatment by actual and simulated doctors. These profiles were then examined by practicing physicians who assessed the "reasonableness" of the exhibited behavior. The results of tests involving many different kinds of therapy indicated the simulation produced highly realistic behavior at the cell level.

It is important to note that the physicians were not asked to evaluate the quality of the treatment given patients by the artificial doctors. They were to evaluate whether or not the simulated behavior was equivalent to behavior generated by similar doctors in comparable real world circumstances. Since the simulation is designed to be an accurate description of the real world environment normative questions are irrelevant.

Once the legitimacy of simulated behavior of the type outlined in Figures 7 and 8 has been established through Turing tests,

the system may be used to produce behavior over time.

Figure 9 illustrates the cumulative prescription market shares generated by two general practitioners operating in the simulated environment during one year. These two doctors prescribed only one relevant drug during the first two weeks of simulated activity. However, as the year progressed, they tried six other drugs. Their cumulative brand shares for the ten brands are shown at week 52. Output of the type illustrated in Figure 9 is used primarily to test system stability.

Population Level Validation

Meaningful tests of population behavior require aggregation of simulated cell behavior. In the doctor case, population behavior is validated by analyzing the proportion of prescriptions allocated to each brand (brand shares), and changes in knowledge, attitudes, and perceived brand images of important segments of the population.

For example, Figure 10 illustrates the brand shares of ten frequently used drugs resulting from 100 simulated doctors' treatment of several thousand patients. In conducting such tests, the population is initialized to duplicate the distribution of relevant parameters as they existed at a specified point in time in the real world environment. In the case of the Figure 10 run, the artificial population was initialized to correspond to conditions existing at the beginning of 1961.

Inputs to the simulation during performance tests describe

conditions existing in the real world during the relevant time period. In this case inputs specified the content and media allocation for all journal, direct mail, salesman detail, and convention promotion generated by competitors operating in the relevant market area during 1961. Tests performed following this simulation run established that the rank order of brand shares at the end of 1961 in the real and simulated worlds were equivalent (Figure 11) and the maximum error for any one brand was less than six percentage points.

It should be pointed out that this test is a duplication of history -- not a prediction of the future. A great deal of real world data was used in providing inputs for this test and in estimating important parameters. When the model is used for prediction of the future, subjective judgment must be used to develop inputs, and assumptions must be made about the stability of important parameters.

MANAGEMENT USES OF MICRO ANALYTIC SIMULATION

Given a system of the type described in this paper, management must assess system performance in terms of intended applications. If, in their opinion, performance is sufficient to warrant use of the simulation as a representation of the real world environment, applications of the type outlined below may be appropriate. However, if, in their opinion, the simulation fails to duplicate salient attributes of the real world environment further development leading to a more refined system must be undertaken or the use of the technique rejected.

Testing Implicit Models

One of the first benefits to accrue from the development of a simulation system is the systematic testing of management conceptions of the environment in which they operate. In reviewing alternative formulations and evaluating functions, cell model behavior, and total population performance, management must make explicit the implicit models which they use in decision making.

The "What If?" Question

Given that management accepts simulation performance as indicative of real world response under comparable conditions, the simulation becomes a test market without a memory in which management may examine with impunity the implications of alternative policies and strategies. Whether introducing new products or considering modification of a marketing program, management may apply alternative strategies in the simulated environment and evaluate their implications under various assumed competitive conditions.

The effectiveness of such pretesting is dependent on management's ability to predict probable competitive responses to proposed actions, as well as the accuracy of the simulation system. Management may find it profitable to examine the impact of best and worst case competitive response patterns. In most instances the best case assumes that competition will continue with programs developed prior to initiation of company actions, while the worst case assumes full competitor

knowledge of the proposed company program and actions designed to thwart company efforts.

Performance References

The simulated environment also provides the reference points against which the progress of operations in the real world may be measured. Given a simulation pre-test, management can determine, by monitoring appropriate variables, whether or not a program is progressing as planned. If conditions producing satisfactory performance in the simulated environment are encountered in the real world, it is assumed that final results will be comparable.

SUMMARY

In this paper we have attempted to discuss the general characteristics of computer simulation, and the process of creating, testing, and using a complex simulation system. In discussing marketing applications we have emphasized complex models and micro analytic simulations since we believe that they best illustrate the differential advantages of the simulation technique.

Finally, the future of simulation in marketing appears to be particularly promising for at least two reasons. First, although systems such as the drug market simulation tax the capacity of the largest commercially available computers, new computers with larger

memories and even greater computational speed are being developed. And second, in spite of the expensive nature of computer simulation, increasing numbers of marketing scholars and executives are coming to agree with the philosophy voiced by a top executive of the firm sponsoring the drug simulation -- "Even if there are significant errors in prediction, it is worth the expense because of the way it makes people think."

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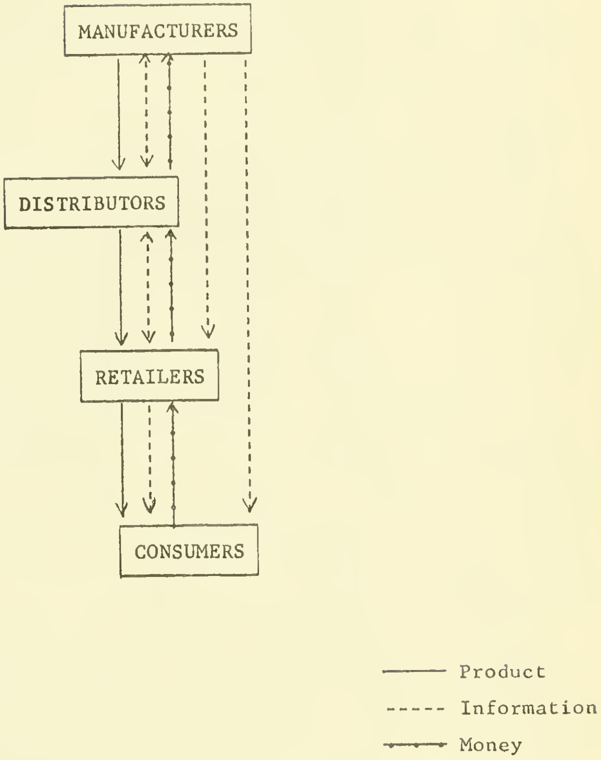
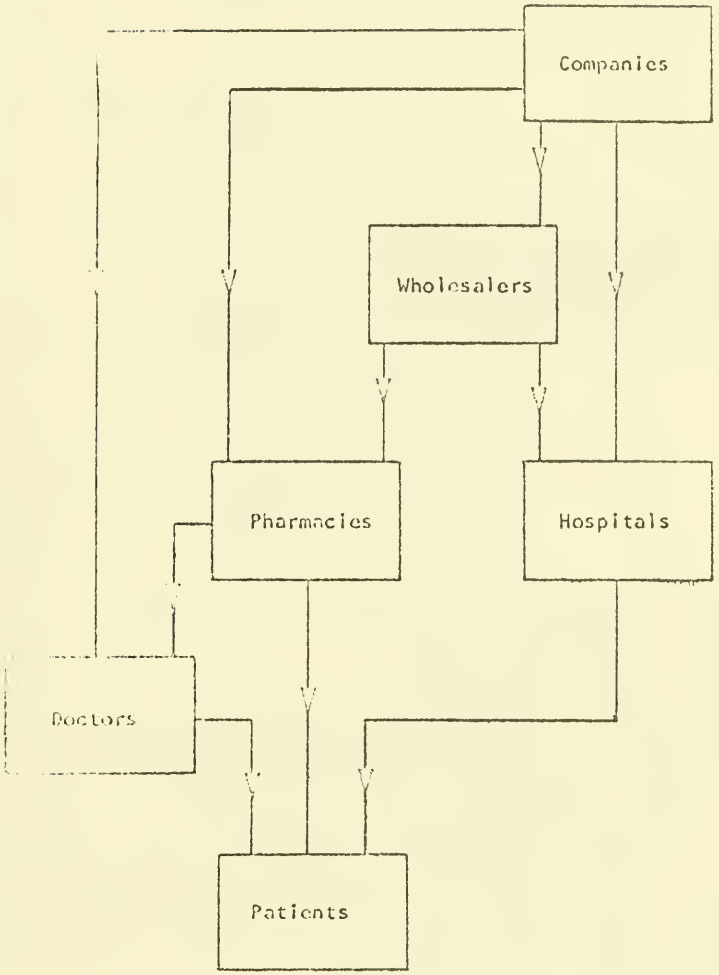


FIGURE 1

A Simplified Representation of
A Consumer Marketing System

Figure 2

BOUNDARY DEFINITIONS -- AN EXAMPLE



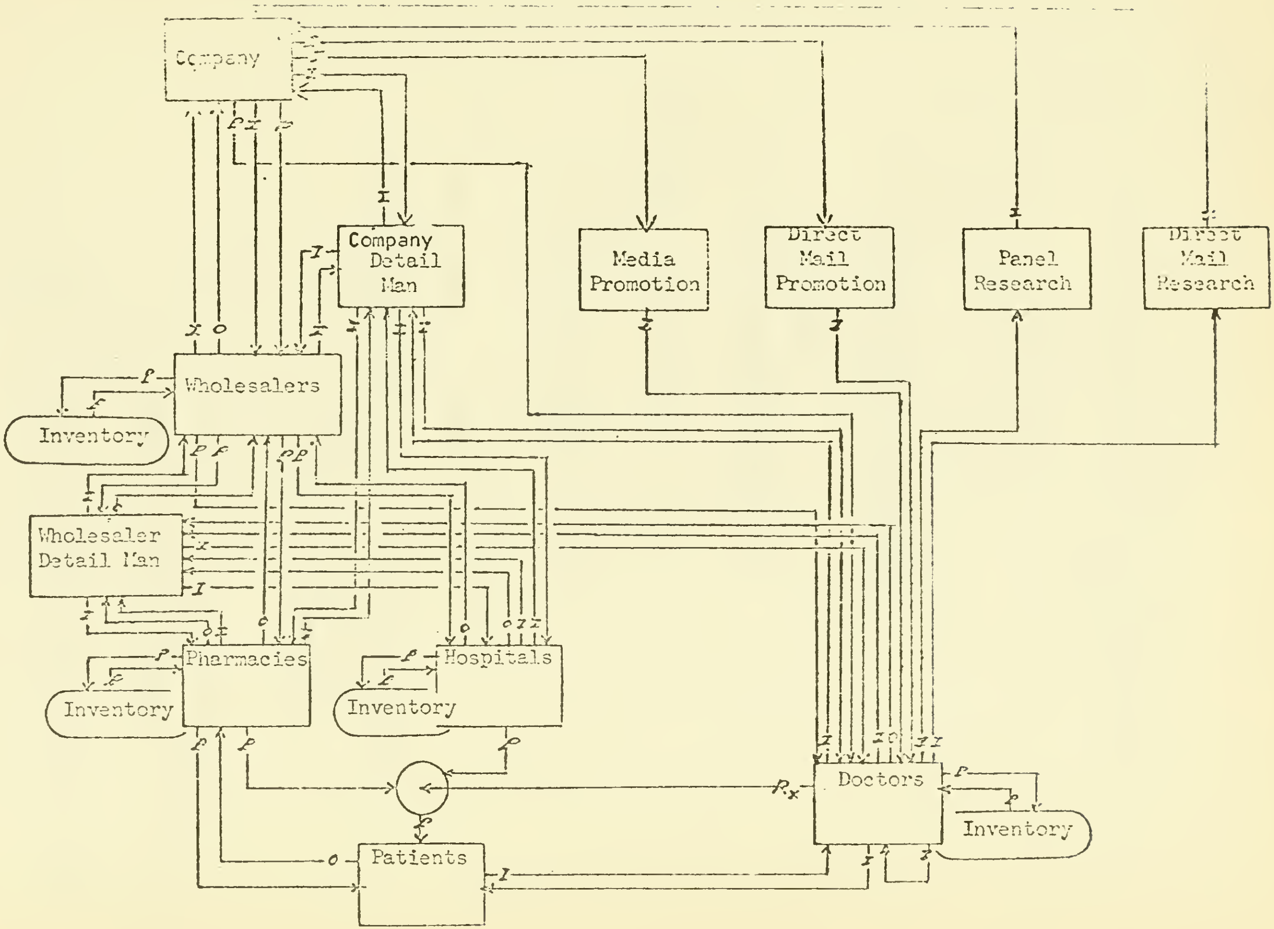


Figure 3

Macro Specifications -- An Example

FIGURE 4

Examples of Data Requirements for Initialization
and Input -- Drug Simulation

1. Specification and Initialization
    - A. Identification of indications
        1. Name
        2. Code number associated with name
        3. Seasonal incidence of indication
            - a. By specialty
            - b. By region
        4. Prioritization of indications
        5. Duration of indications
    - B. Identification of drugs
        1. Name
        2. Code number associated with name
        3. Classification code
        4. Manufacturer code
    - C. Initialization information for drugs
        1. Drug code number
        2. Uses of specific drugs for specific indications
            - a. Number of appearances by Doctor type
            - b. Success ratios in treating each indication
        3. Existing promoted brand image
            - a. Print media utilization during preceding year
                - 1) Media identification by code number
                - 2) Number of full page equivalents run in each medium
                - 3) Content
            - b. Description of direct mail promotion during preceding year -- as in 'a' above

Figure 4 (continued)

- c. Description of detail program during preceding year
    - 1) Number of product details during year
    - 2) Number of sample calls during year
    - 3) Content

D. Identification of distribution channels

- 1. Major wholesalers
    - a. Territories covered
    - b. Relevant drugs handled
- 2. Major drug outlets as in 1 above
- 3. Major hospitals as in 1 above

E. Identification of company salesman

- 1. Territory code
- 2. Average number of calls per day to doctors, hospitals, and pharmacies

F. Identification of competitive companies

- 1. Company name
- 2. Manufacturer code
- 3. Number of salesmen
- 4. Average number of calls per day to doctors, hospitals, and pharmacies

G. Indication - action - appeals matrix

- 1. By indication
- 2. By specialty

H. Number of doctors by type

- 1. Drug usage record
- 2. Media availabilities
- 3. Proportion of practice devoted to treatment of indications
- 4. Patient load, etc.

Figure 4 (continued)

II. Input

A. Variance in incidence of indication

1. By indication
2. By time period

B. Media advertising -- specified for each advertisement

1. Date of release
2. Media code
3. Drug promoted
4. Ad format
5. Ad content

   a. Appeals
   b. Actions
   c. Indications

C. Direct mail specification

1. Date of mailing
2. Region mailed
3. Proportion of each specialty mailed
4. Content - as in B-5 above

D. Company salesman detail

1. Date of release
2. Drug promoted
3. Priority
4. Content of detail - as in B-5 above

E. Company sample handling

1. Drug code
2. Sample size
3. Number of samples
4. Proportion of specialty covered
5. Territories sampled
6. Date of sampling

Figure 4 (continued)

F. Convention presentations

    1. Date
    2. Product presented
    3. Attendance

        a. Regions from which attendance drawn
        b. Specialty from which attendance drawn

    4. Content of presentation - as in B-5 above

G. Symposia

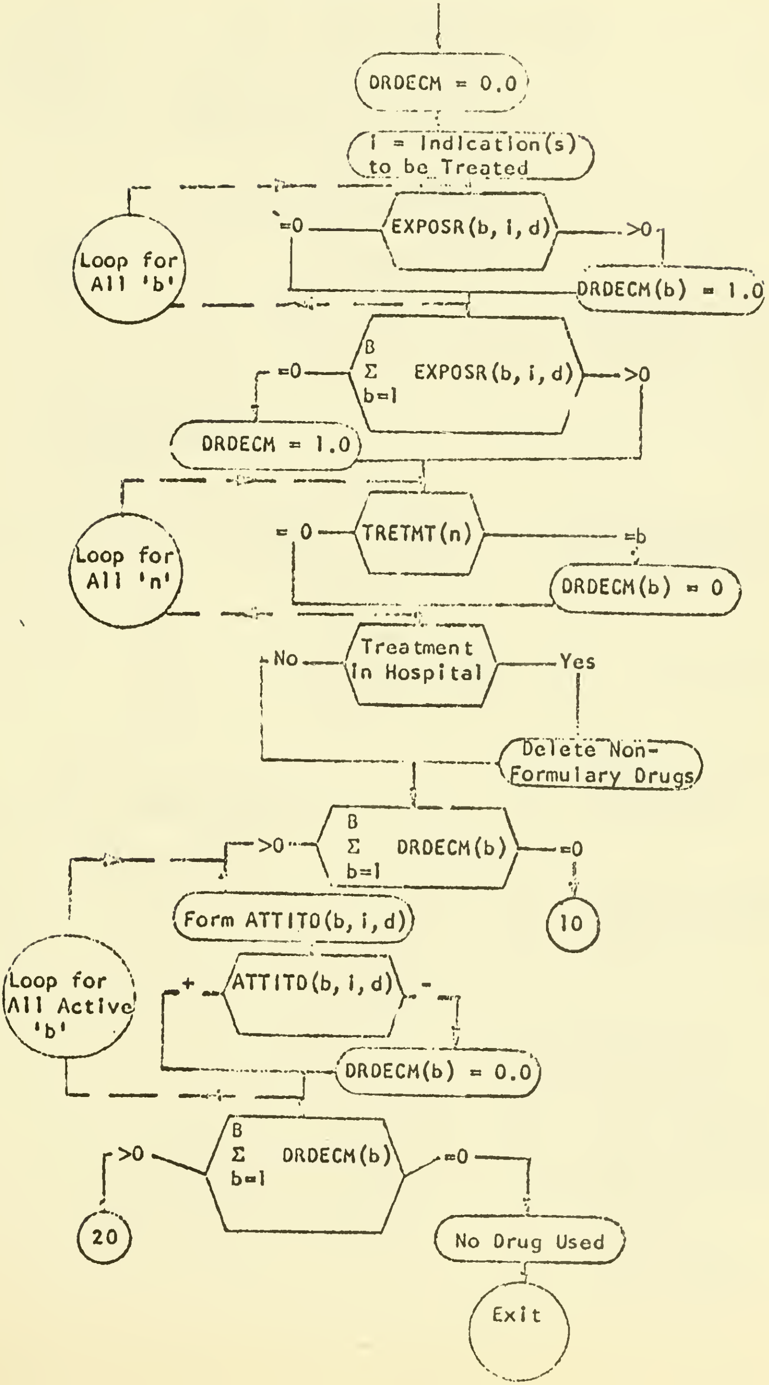
- Handled as conventions

H. Public relations releases

- Handled as advertising

Figure 5

DRUG SELECTION PROCEDURE



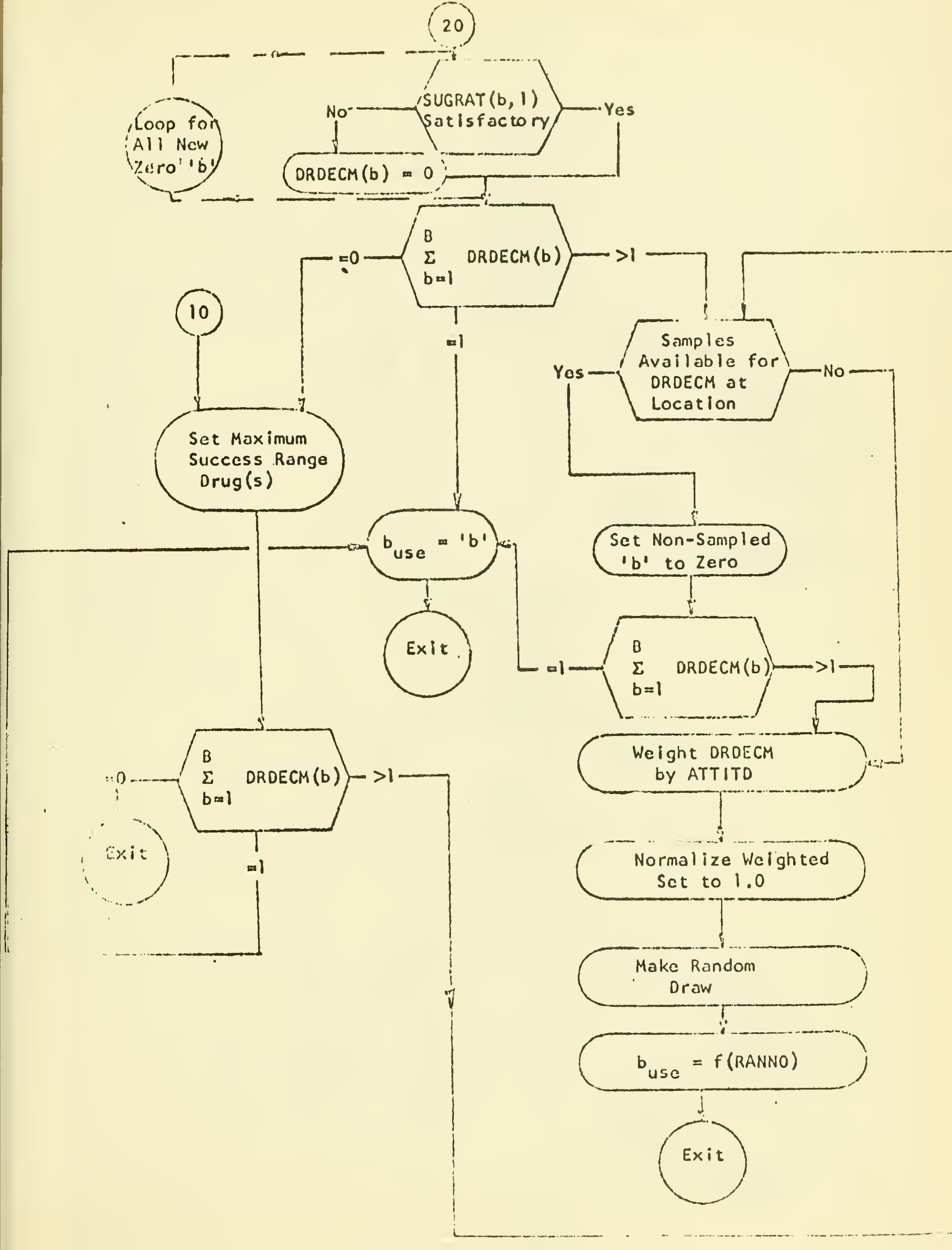

20
Loop for All New Zero 'b'
SUGRAT(b,1) Satisfactory
No
Yes
DRDECM(b) = 0
$\sum_{b=1}^{B}$ DRDECM(b)
=0
>1
=1
10
Samples Available for DRDECM at Location
Yes
No
Set Maximum Success Range Drug(s)
b_{use} = 'b'
Set Non-Sampled 'b' to Zero
Exit
$\sum_{b=1}^{B}$ DRDECM(b)
=1
>1
Weight DRDECM by ATTITD
$\sum_{b=1}^{B}$ DRDECM(b)
=0
>1
=1
Exit
Normalize Weighted Set to 1.0
Make Random Draw
b_{use} = f(RANNO)
Exit

Figure 6

MACRO FLOW CHART OF SYSTEM OPERATION

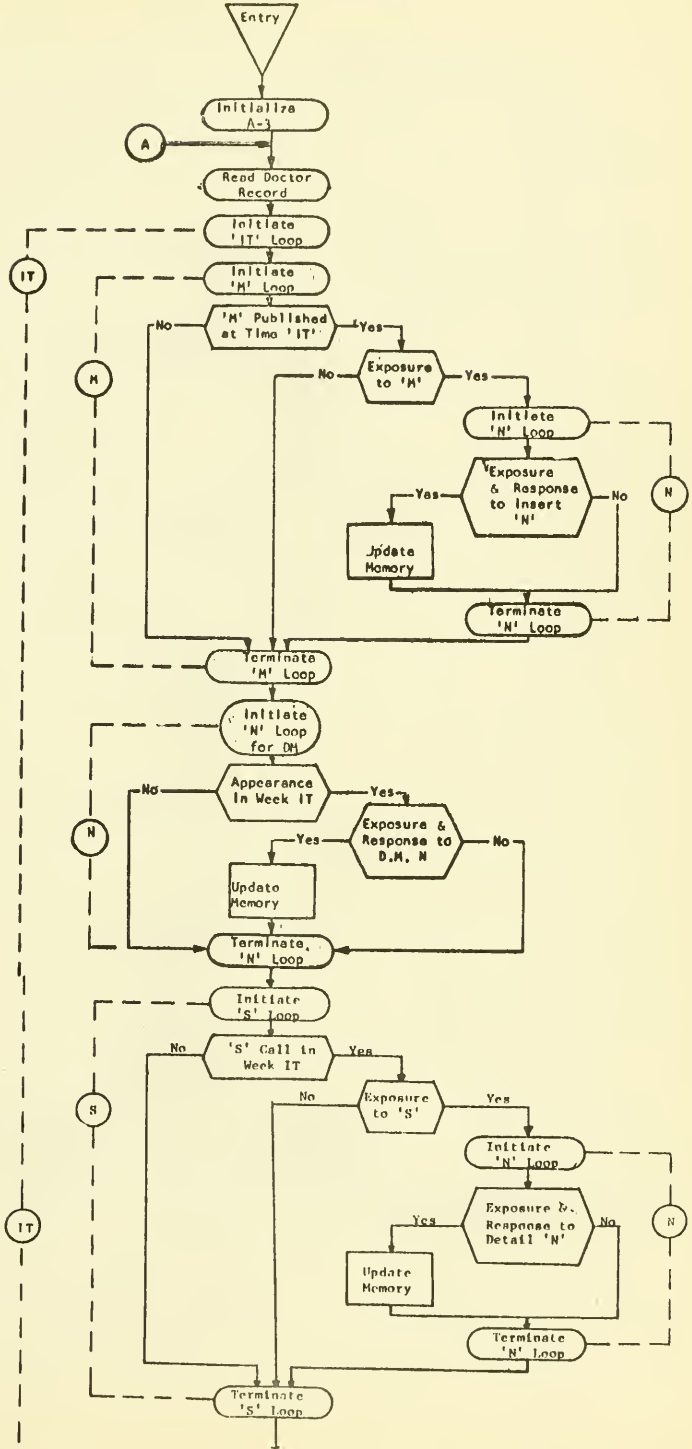
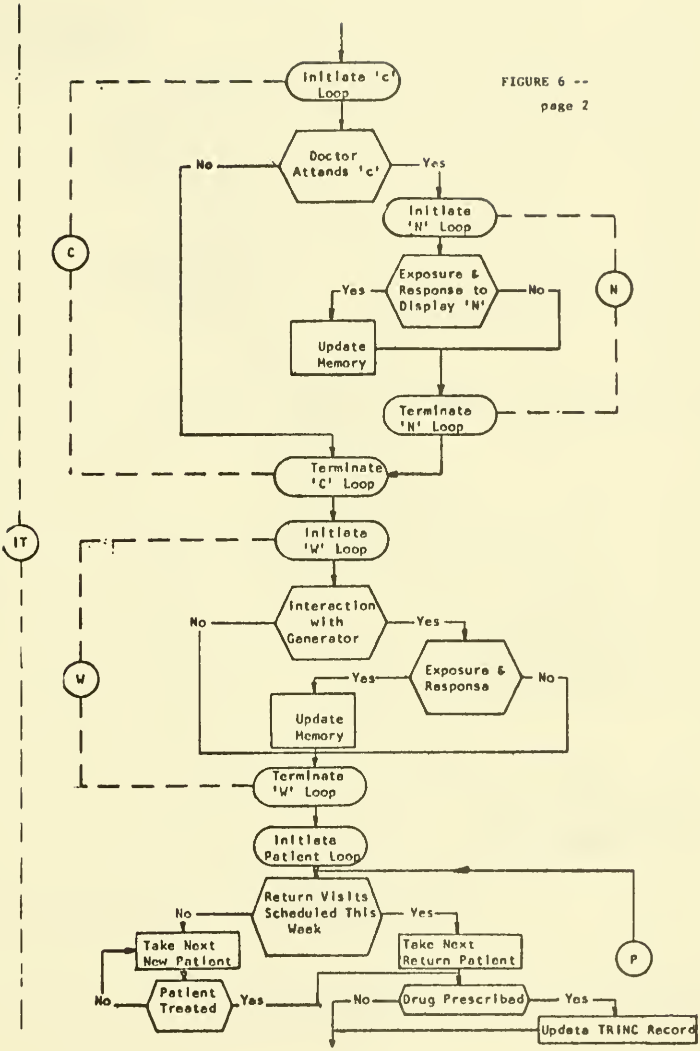


FIGURE 6 --

page 2



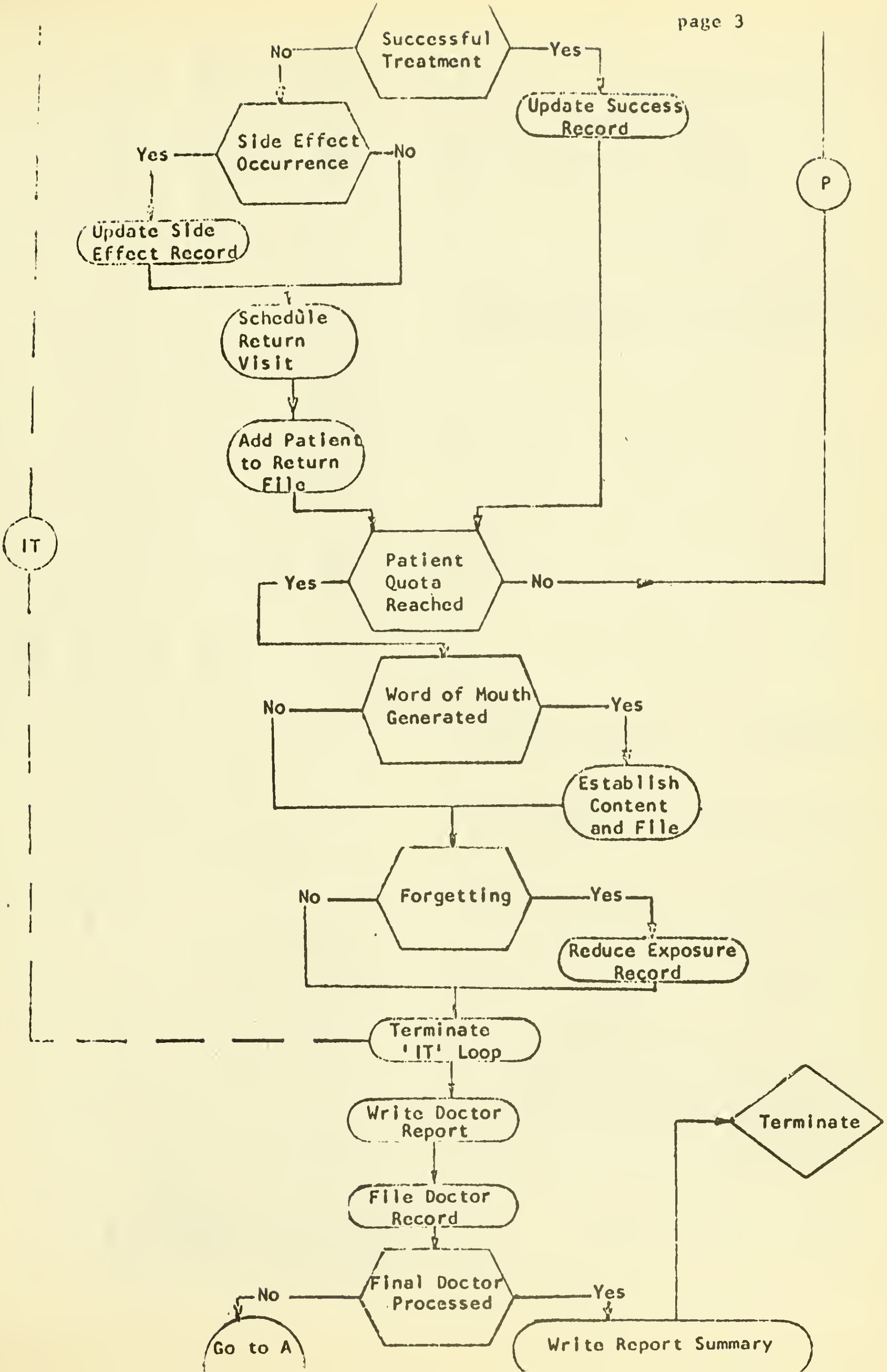

Successful Treatment
No
Yes
Update Success Record
Side Effect Occurrence
Yes
No
Update Side Effect Record
P
Schedule Return Visit
Add Patient to Return File
IT
Patient Quota Reached
Yes
No
Word of Mouth Generated
No
Yes
Establish Content and File
Forgetting
No
Yes
Reduce Exposure Record
Terminate 'IT' Loop
Write Doctor Report
File Doctor Record
Final Doctor Processed
No
Yes
Go to A
Write Report Summary
Terminate

FIGURE 7

```
                              TURING TEST RE MULTIPLE DRUG USE ---MARCH 26, 1966
DOCTOR      1, OF SPECIALTY  1, S.M.A. 1 IN NDTI REGION 2, PRACTICE TYPE 1, AGE CAT. 3 IS NOW BEGINNING

AVERAGE PATIENTS TREATED PER WEEK 136 SCREENING EMPLOYEE INDICATOR 1

CIRCULATION PROBABILITIES
 1.0000  0.5750  0.3050  0.9480  1.0000  0.9220  1.0000  0.5030  0.0440  0.9410  0.9510  0.9080  1.0000

MEDIA-EXPOSURE-PROBABILITES
 0.2280  0.1640  0.1280  0.2670  0.3620  0.2560  0.1510  0.1410  0.1220  0.2520  0.3080  0.1130  0.1850

COMPANY EXPOSURE PROBABILITIES
0.2700 0.2000 0.1400 0.1000 0.        0.2000 0.1400 0.1600 0.0200 0.4800 0.5300 0.1500 0.1300 0.1800 0.450
0.3900 0.3700 0.3100 0.1900 0.1600 0.1300 0.        0.1800 0.3300 0.3300 0.0800 0.4700 0.        0.1400 0.170
OR-WEIGHTING    1



SEASONAL PROPORTION OF PRACTICE TREATING INDICATIONS OF INTEREST , DOCTOR'S PROBABILITY OF USING A SIMU
            1 TB       .0009  .0007  .0012  .0010                                          0.
            2 SYPLIS   .0004  .0000  .0004  .0009                                          0.0446
            3 GONORR   .0009  .0011  .0010  .0014                                          0.
            4 OT-VED   .      .0001  .0001  .0002                                          0.
            5 INFINT   .0180  .0200  .0113  .0215                                          0.0197
            6 SCAFVR   .0001  .0001  .0002  .                                              0.
            7 STRPST   .0065  .0008  .0023  .0043                                          0.
            8 CT-RAD   .0004  .0000  .      .0003                                          0.
            9 VININF   .0002  .0000  .0003  .0002                                          0.
           10 SPIRXS   .      .      .      .                                              0.
           11 POLIO    .      .      .0000  .                                              0.
           12 MESLES   .0029  .0023  .0015  .                                              0.
           13 GERMES   .      .0002  .0004  .0004                                          0.
           14 CHIKPX   .0009  .0010  .0005  .                                              0.
           15 HERZOS   .0014  .0016  .0024  .0017                                          0.0173
           16 MUMPS    .0013  .0002  .0005  .0004                                          0.
           17 INFHEP   .0005  .0011  .0005  .0006                                          0.0117
           18 GLANFV   .0000  .0003  .0005  .0006                                          0.
           19 OT-VID   .0027  .0020  .0023  .0008                                          0.0024
           20 M-T-RD   .      .      .0001  .0001                                          0.
```

FIGURE 8

Patient Treatment

```
PATIENT    1 IS A GERIATRIC,SEEN  1 TIMES PREVIOUSLY AND GIVEN APPOINTMENT IN WEEK  5
PATIENT EXHIBITS INCICATIONS URI     WITH  0 ACTIONS DESIRED ,        ,
                                 CONDITION IMPROVING.
                                 ANXREA WITH  2 ACTIONS DESIRED , ADEPRE, TNCGEN,
                                 CONDITION UNCHANGED.
PRESENT TREATMENT DRUGS NONSIM, TOFRAN,         . NO PRIOR TREATMENT.
DRUG    1 EQUANL NOTED AS ASSOCIATED WITH INDICATION  47 ANXREA
DRUG    2 LIBRUM NOTED AS ASSOCIATED WITH INDICATION  47 ANXREA
DRUG    3 MILTWN NOTED AS ASSOCIATED WITH INDICATION  47 ANXREA
DRUG    6 MARPLN NOTED AS ASSOCIATED WITH INDICATION  47 ANXREA
DRUG    8 RITALN NOTED AS ASSOCIATED WITH INDICATION  47 ANXREA
DRUG    9 TOFRAN NOTED AS ASSOCIATED WITH INDICATION  47 ANXREA
DRUG    9 TOFRAN DELETED DUE TO PRIOR TREATMENT
DRUG  1 EQUANL ATTITUDE FORMED =          3 FOR IND. 80 URI    AND  47 ANXREA
DRUG  2 LIBRUM ATTITUDE FORMED =          2 FOR IND. 80 URI    AND  47 ANXREA
DRUG  3 MILTWN ATTITUDE FORMED =          0 FOR IND. 80 URI    AND  47 ANXREA
DRUG  6 MARPLN ATTITUDE FORMED =          2 FOR IND. 80 URI    AND  47 ANXREA
DRUG  8 RITALN ATTITUDE FORMED =          0 FOR IND. 80 URI    AND  47 ANXREA
DRUG    3 MILTWN DELETED DUE TO NEG. OR ZERO ATD.
DRUG    8 RITALN DELETED DUE TO NEG. OR ZERO ATD.



DETERMINING HIGHEST CRITERIA DRUG, IPATH = 1, ICASE = 1
DRUG  1 EQUANL, CRITERIA VALUE =  3.000
DRUG  2 LIBRUM, CRITERIA VALUE =  2.000
DRUG  6 MARPLN, CRITERIA VALUE =  2.000
 1 UNSATISFIED ACTIONS  TNCGEN
DRUG  2 LIBRUM ATTITUDE FORMED =         0 FOR UNSATISFIED ACTIONS
DRUG  3 MILTWN ATTITUDE FORMED =         0 FOR UNSATISFIED ACTIONS
DRUG  4 STELAZ ATTITUDE FORMED =         0 FOR UNSATISFIED ACTIONS
DRUG  5 THORAZ ATTITUDE FORMED =         0 FOR UNSATISFIED ACTIONS
DRUG  6 MARPLN ATTITUDE FORMED =         0 FOR UNSATISFIED ACTIONS
DRUG  7 PARNAT ATTITUDE FORMED =         0 FOR UNSATISFIED ACTIONS
DRUG  8 RITALN ATTITUDE FORMED =         0 FOR UNSATISFIED ACTIONS
DRUG 10 NOLDAR ATTITUDE FORMED =         0 FOR UNSATISFIED ACTIONS
DRUG 11 PLACOL ATTITUDE FORMED =         0 FOR UNSATISFIED ACTIONS
NO DRUG FOR UNSATISFIED ACTIONS
EVALUATION COMPLETED, TEMP SET   0  0   1  0   0  0 ID1        ID2 EQUANL ID3
FATE, RETURN  0, SUCCESS 1 1 0,  S.E. 0 0 0
```

FIGURE 9

Sample Output -- Two Doctors

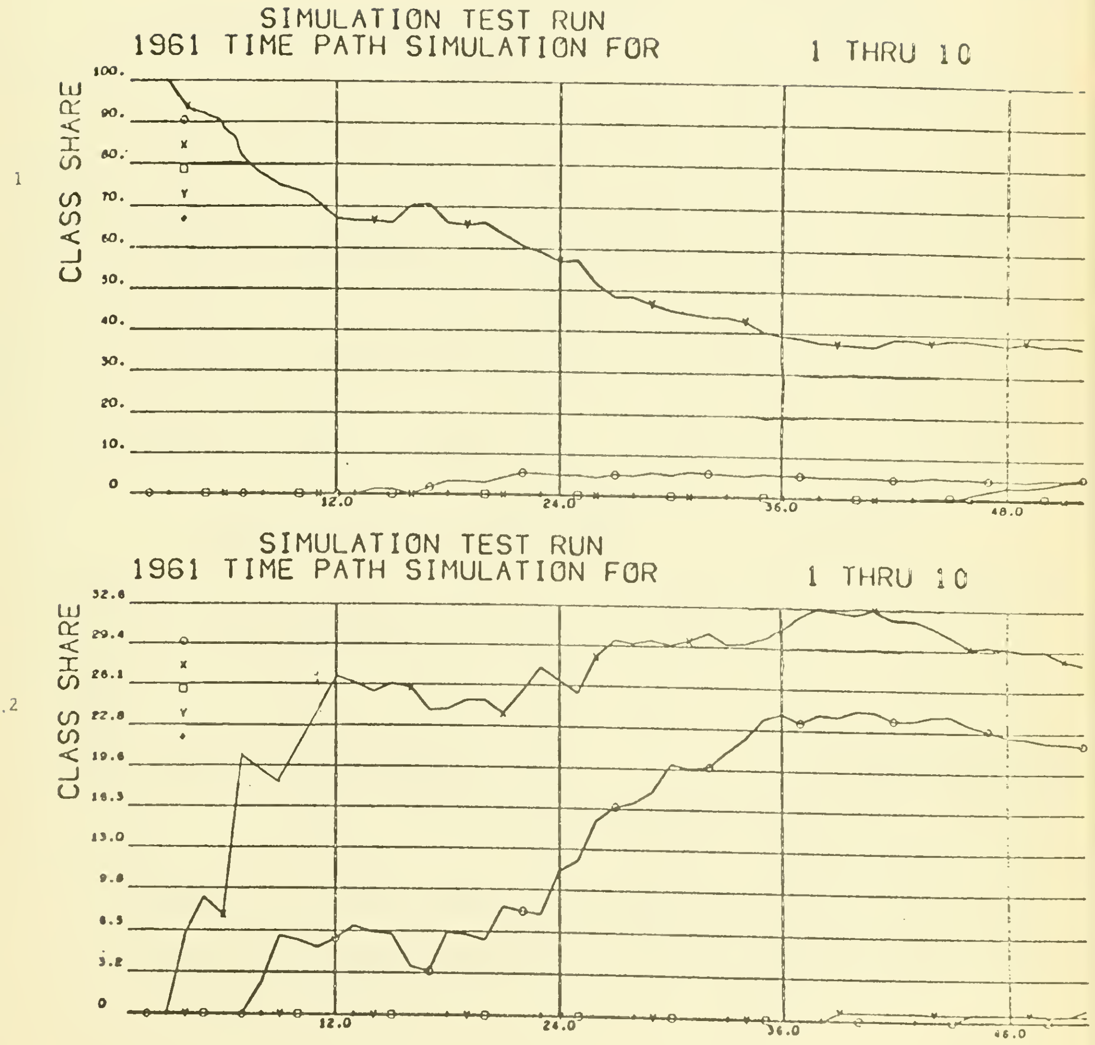
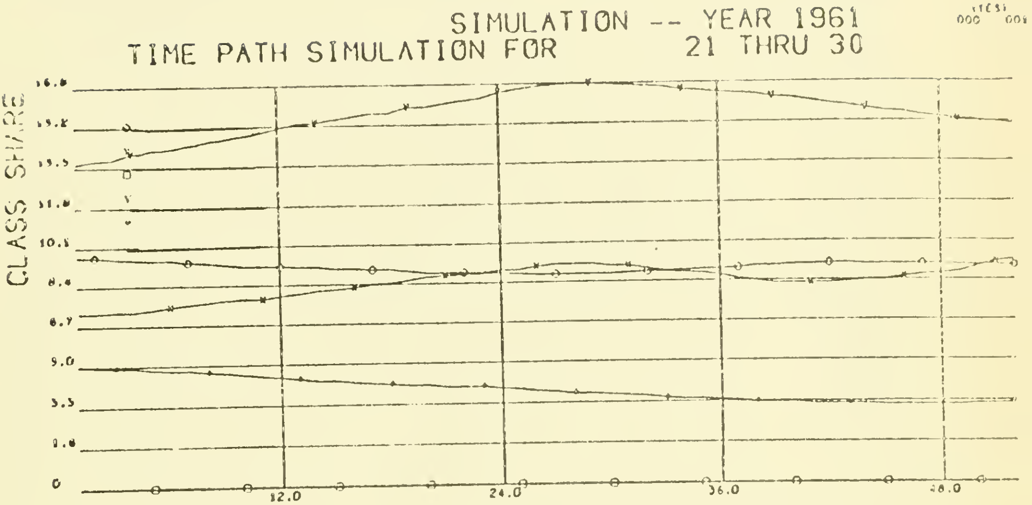


FIGURE 10

Sample Output -- 100 Doctors

1



2

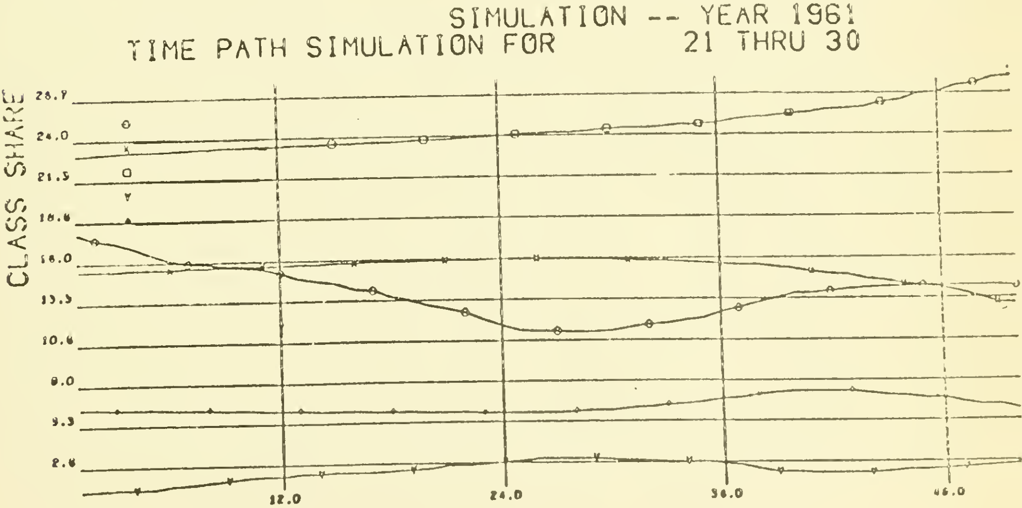


FIGURE 11

Rank Order Brand Share Comparisons

Identification	Rank as Initialized	Year End Rank Simulated	Year End Rank Actual
1 - Y	4	2	2
1 - 0	5	6	6
1 - X	6	5	5
1 - +	8	8	8
1 - ☐	10	10	10
2 - ☐	1	1	1
2 - 0	2	3	3
2 - X	3	4	4
2 - +	7	7	7
2 - Y	9	9	9

A GEOGRAPHIC MODEL OF AN URBAN AUTOMOBILE MARKET

Theodore E. Hlavac, Jr.

John D. C. Little

180-66

April 1966

Abstract

A person chooses a store to shop at partly on the basis of the difficulty of getting there. An understanding of the relationships involved is important in marketing strategy, particularly for site selection purposes. We study an urban new-car market. As a measure of shopping difficulty, the distance from a person's residence to the car dealer is used. Assumptions about buyer behavior lead to a model of competitive interaction among dealers and car makes. The model is fitted to three months of sales data for metropolitan Chicago. An interactive computer system has been programmed to make the model available for on-line investigation of a variety of site selection and inter-brand competition questions. For example, a user can add, eliminate, or move a dealer and then ask for the model-predicted changes in the sales and penetration of any dealer or make.

A GEOGRAPHIC MODEL OF AN URBAN AUTOMOBILE MARKET

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Sloan School of Management
Massachusetts Institute of Technology

1. Introduction

When a person buys a car, he takes into account either explicitly or implicitly, the distance he must travel to the prospective place of purchase. He is likely to be attracted to a nearby dealer because of easy accessibility for shopping trips and future service visits. He is less likely to be attracted to a distant dealer, but some attraction will exist, especially in the case of a well-advertised dealer having an image of low price and high throughput. Within an urban market, where there is considerable choice of dealers, the probability of purchase from a given dealer can be expected to fall off with distance; and, in fact, such a relationship is easily established empirically.

In the present paper we develop a model in which a customer's probability of purchase at a given dealer is affected by dealer location and customer make preference, as well as the locations and strengths of all other dealers. Aggregation of the customer model gives a dealer market share (penetration) model, which may also be viewed as a model of competitive interaction. Such a model is fit to data for metropolitan Chicago. After fitting, the model permits estimation of the sales of a dealership with specified strength and location.

The most obvious practical use of the model relates to market strategy for new dealerships in the automobile industry, but the model appears to be adaptable to site location problems in other fields as well.

This work was supported in part by Project MAC, an MIT research program sponsored by the Advanced Research Projects Agency, Department of Defense, under Office of Naval Research Contract Number Nonr-4102(01). Part of the research was done at the MIT Computation Center.

2. Model

2.1 Dealer Pull. To some extent all dealers are attractive to a person planning to buy a car. The attractiveness of a specified dealer presumably is a function of dealer characteristics (such as make of car sold, extent of advertising), buyer characteristics (such as make preference), and distance from dealer to buyer. The attractiveness of a dealer will be called his "pull". Pull will not be a directly observable quantity but will be used to develop expressions that are.

We shall assume that the number of car purchases is fixed for the time period under consideration. We have not modeled the effect of a dealer in creating sales that would not have occurred in his absence. Our data do not seem to lend themselves to the estimation of this effect, and perhaps it is small in today's well developed markets.

Buyers will be separated into market segments. The pull of a dealer on a buyer in a given segment will be broken into two parts: (1) an intrinsic pull independent of the make sold by the dealer and (2) the make preference of the buyer. Let

$g(i,j)$ = the pull of dealer j on a buyer in market segment i. $i = 1,\dots,S$ $j = 1,\dots,D$.

$h(i,j)$ = the intrinsic pull of dealer j on a buyer in segment i.

$q(i,m)$ = the make preference of a buyer in segment i for make m. $m = 1,\dots,M$. We specify that $q(i,m) \geq 0$ and $\Sigma_{m=1}^{M} q(i,m) = 1$.

Let m(j) denote the make sold by dealer j. We stipulate that the above quantities be related by

$$g(i,j) = h(i,j)\, q(i, m(j)) \qquad (1)$$

Thus, the pull of a dealer on a buyer is the dealer's intrinsic pull weighted by the buyer's brand preference.

2.2 Purchase probability. The probability that a buyer purchases at a given dealer will be taken as the pull of that dealer on the buyer divided by the total pull on the buyer. Let

$p(i,j)$ = the probability that a buyer in market segment i purchases at dealer j.

$$p(i,j) = g(i,j) \,/\, \Sigma_{k=1}^{S} g(i,k) \qquad (2)$$

We note that make preference can be interpreted as the probability of purchase of the make under the conditions that the sum of the intrinsic pulls on the buyer is the same for each make. This result can be deduced from (1) and (2).

2.3 Geographic effect. We hypothesize that pull falls off exponentially with the distance between dealer and buyer. This is only one of many possible relations but it appears to work well. Let

$x(i,j)$ = distance of the buyers in market segment i to dealer j.

$$h(i,j) = a_j e^{-b_j x(i,j)} \tag{3}$$

Here a_j and b_j are constants specific to dealer j. The constant a_j expresses the dealer's strength in his own immediate neighborhood. The constant b_j tells how fast his sales fall off with distance. Using (1), (2), and (3), we get

$$p(i,j) = \frac{q(i,m(j))\ a_j\ e^{-b_j x(i,j)}}{\Sigma_{k=1}^{D}\ q(i,\ m(k))\ a_k\ e^{-b_k\ x(i,k)}} \tag{4}$$

2.4 Dealer sales and penetration. Let

$N(i)$ = number of buyers in market segment i (called the potential of the segment) in a given time period.

$s(j)$ = expected sales of dealer j in the given time period.

$\pi(j)$ = expected penetration of dealer j in the whole city.

Then

$$s(j) = \Sigma_{i=1}^{S}\ N(i)\ p(i,j) \tag{5}$$

$$\pi(j) = s(j)\ /\ \Sigma_{i=1}^{S}\ N(i) \tag{6}$$

3. Fitting the Model to Data

3.1 Data. The model has been fit to R.L.Polk new car registrations for April, May, and June 1963 in Cook County, Illinois (fleet sales eliminated). In order to maintain reasonable sample sizes with three months of data, the analysis has been confined to 8 major makes (Ford, Chevrolet, Rambler, Pontiac, Plymouth, Buick, Oldsmobile, and Dodge) and to dealers selling 80 or more cars in the time period. This involves 47,670 cars and 184 dealers, or 91.5% of the cars and 62% of the dealers of the makes

considered. The county has been divided into 140 marketing areas or cells. The rationale for this is discussed by James [1]. The location of each dealer has been determined as has been the sales of each dealer in each cell.

The market has been segmented by geographic area, i.e., by cell. For make preference we have used market share. Since the city is laid out rectangularly, distance has been measured rectangularly instead of diagonally. Some alternatives to these choices will be suggested below.

3.2 Fitting procedure. The model has been fit to the data by a modified maximum likelihood procedure. We observe that, if the denominator of (4) is regarded as a known constant, each dealer becomes a separate estimation problem involving two parameters, a_j and b_j, and 140 data points, the dealer's sales in each cell. Given all the a_j and b_j the denominator of (4) can be calculated. This suggests an iterative procedure. Suppressing the dealer index j, let

n_i = sales of dealer j in cell i.

$$w_i = q(i,\ m(j))/\Sigma_{k=1}^{D}\ q(i,\ m(k))\ a_k\ e^{-b_k\ x(i,k)} \qquad (7)$$

$p_i = w_i\ a\ e^{-bx_i}$ = probability a buyer in cell i purchases at dealer j.

Then assuming independence of purchase, the likelihood function for the observed sales figures is

$$L = \pi_{i=1}^{S}\ \binom{N(i)}{n_i}\, p_i^{\,n_i}(1-p_i)^{N(i)-n_i} \qquad (8)$$

We start with a trial set of w_i. Values of a and b are then chosen to maximize L. This is done by setting the derivatives of log L with respect to a and b to zero and solving the resulting equations by Newton's method. The a and b are calculated for each dealer. These are then used to recompute the w_i from (7). The process is repeated until the values of the a's and b's converge.

3.3 Results. Figure 1 shows the difference between actual sales and model-predicted sales for the 184 dealers. The two are very close. This is of course desirable, but it is only a moderately good measure of model fit. Although the fitting is done on cell sales rather than dealer sales, and there is no requirement that dealer sales fit exactly, it would be expected from the nature of the calculation that the fit would be close. Figure 2 compares actual and predicted penetration by cell for one dealer. The discrepancies are considerable but, again, they are only a fair indication of fit. Most of these discrepancies are random fluctuations resulting from the small sample sizes involved in any given cell.

Figure 1.

Error (Predicted Sales - Actual Sales) vs. Dealer Size (Actual Sales)

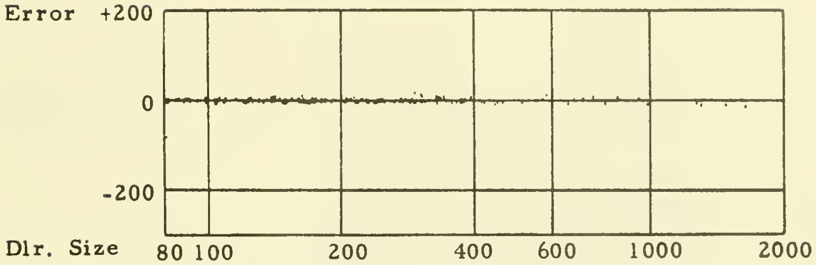
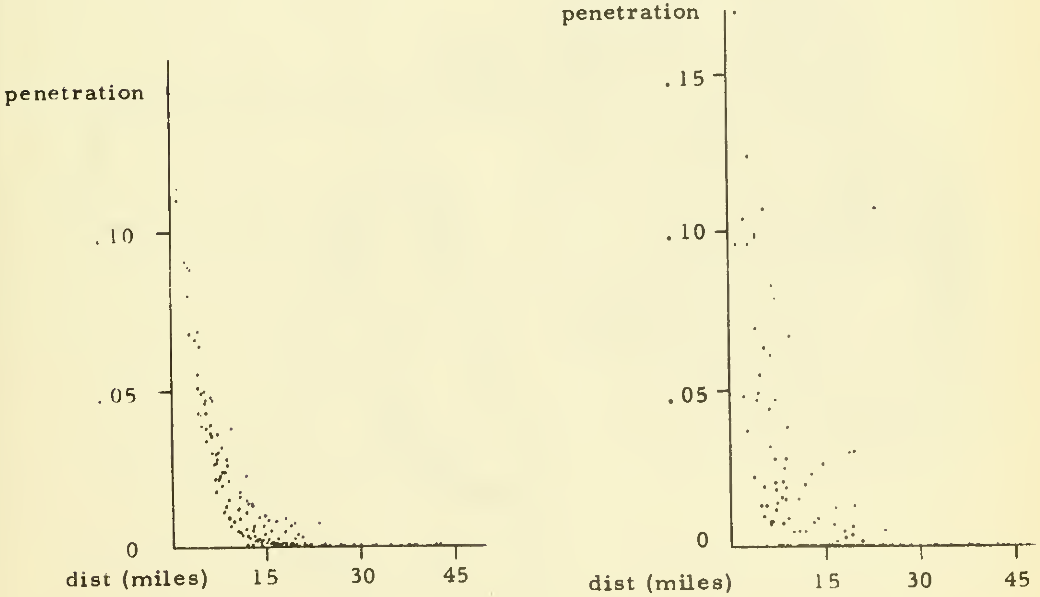


Figure 2.

Penetration vs. Distance from Dealer for a Large Chevrolet Dealer .

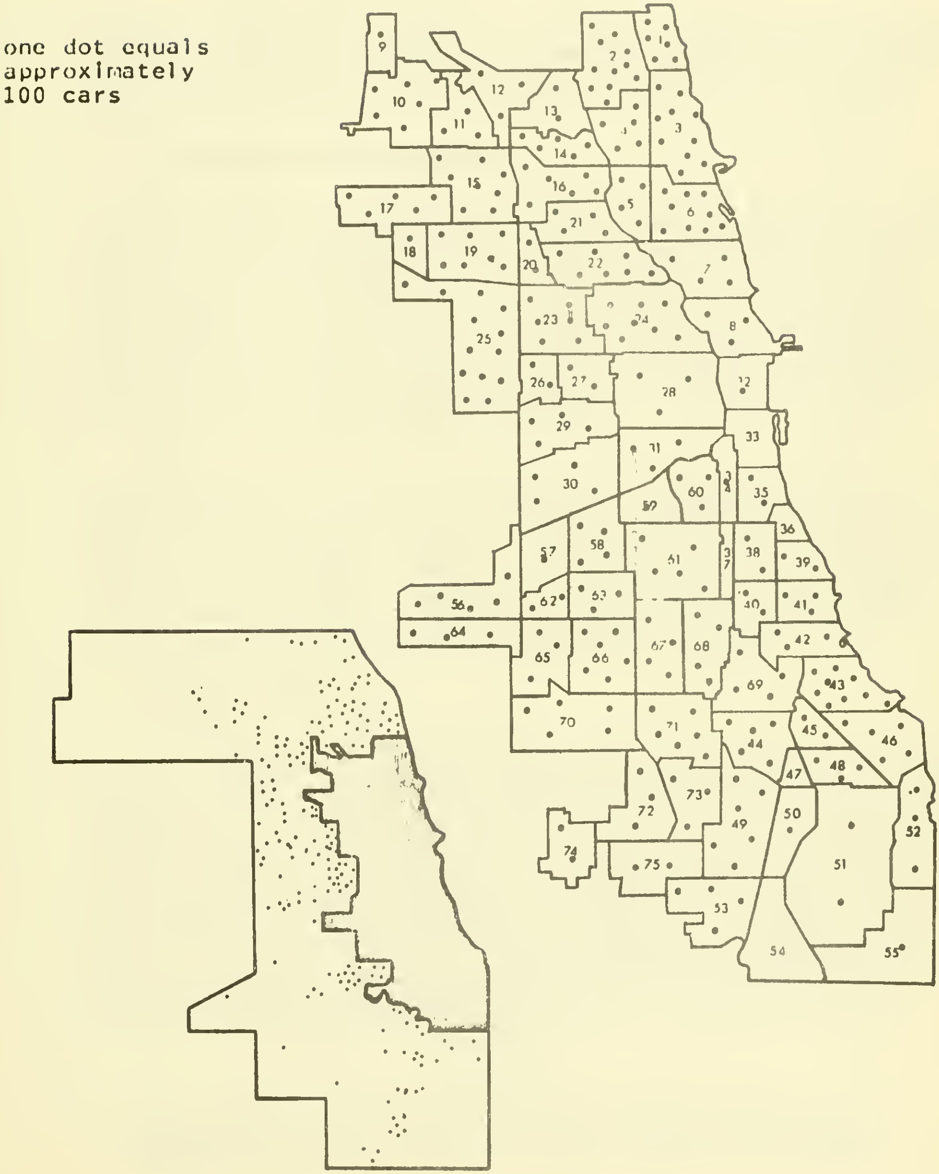


2a. Predicted Penetration vs. Distance

2b. Actual Penetration vs. Distance

Figure 3.

POTENTIAL DENSITY



one dot equals approximately 100 cars

For a better appraisal of fit, we have hypothesized that, for a given dealer, the probability of purchase in cell i is not p_i but is p_i' where

$$p_i' = p_i(1 + \epsilon_i)$$

and ϵ_i is a random error term. The standard deviation, σ of ϵ_i may be viewed as a type of a coefficient of variation. The value of σ has been estimated for each dealer and has a median value of .32. This seems quite good, considering how few factors are included in the model.

4. An Interactive Computer System

Conceivably, one could use the model to work out a mathematically optimal pattern of dealers over the city or, more modestly, the optimal location of a new dealer. Such an optimization is probably sterile. A decision on a dealership involves many factors not included in the model: the availability of property, financing, the micro-geography of the location, etc. Perhaps some of these factors can be modeled but as of now they are not. Yet we do not want to wait to take advantage of what we can learn about macro-geography and competitive interaction. What is needed is a convenient way to make the information available to a person working on dealership problems.

To demonstrate how this can be done, an interactive computer system was programmed for the model on the Project MAC time-shared computer at MIT. With the system a user can sit at a remote console of the computer, make hypothetical changes in dealerships, and learn immediately the model's prediction of the effects. See Exhibit I for an example of the system in action. Changes that a user can make include adding a dealer, moving a dealer, eliminating a dealer, changing a dealer's a and b parameters, and changing the potential of a cell.

5. Possible Improvements

Experimentation with the functional form of the distance relations would be of interest. Similarly, different measures of "distance" might be tried. Travel time has strong intuitive appeal. Much better market segmentation is possible. The natural one would be to break down the cell population by make-model year of car owned. Then make preference could be related to the buying rates of each segment for each make. Another subject for investigation is the effect of clustering of dealers. Perhaps there is a special advantage to being in an "automobile row" because the row itself attracts customers. If so, the model might be modified to account for this. A highly desirable line of research would be to investigate how the a and b parameters are related to various observable characteristics of the dealer.

Reference

1. J. W. James, "An Analysis of the Optimum Market Representation Policies Relating to a Large Metropolitan Market," MS Thesis, MIT, 1964.

EXHIBIT I
A TYPICAL CONSOLE SESSION

```
AN INTERACTIVE MODEL OF THE                         -the computer prints a title
CHICAGO AUTOMOBILE MARKET                            (User commands are always in lower
                                                     case, computer responses in upper
T. E. HLAVAC, JR.                                    case)
J. D. C. LITTLE

BEGIN PLEASE                                        -a signal that initialization is
                                                     complete and the user may begin
move dlr 222 to cell 131                            -the user moves a dealer from his
                                                     present location

 DODG DEALER NUMBER 222 HAS BEEN MOVED TO CELL 131. -the system executes the instruction,


CONTINUE                                             and requests the user to continue

change for dlr 222                                  -the user wishes to know how this move
                                                     affects the dealer's performance

 DODG DLR 222         SALES PENETRATION
 FORMER SYSTEM          82       .17126
 PRESENT SYSTEM         73       .15391             -the dealer suffered a net loss by
 NET LOSS                9       .01735              the move,


CONTINUE

begin                                               -so the user returns the system to
                                                     its initial state


CONTINUE                                            -ready for a fresh start


potential of cell 54, pot = 600                     -the user wishes to increase the
                                                     potential of a cell, possibly because
                                                     of an anticipated new housing
THE POTENTIAL OF CELL  54 HAS BEEN                   development
CHANGED FROM    8 UNITS TO  600 UNITS.
THE TOTAL POTENTIAL OF THE SYSTEM
IS NOW 48262 UNITS.                                 -the result of the change


CONTINUE

change in dodg sales                                -how did the increase in potential
                                                     affect Dodge sales?

  8 DODG DEALERS      SALES  PENETRATION
 FORMER SYSTEM        1119     2.34671
 PRESENT SYSTEM       1128     2.33794              -Dodge dealers obtained 9 of the
 NET GAIN                9     -.00877               592 new units


CONTINUE

add dodg in cell 54, a = 4.0, b = -.32              -an additional Dodge dealer is added
                                                     in the newly developed cell

THIS DEALER HAS BEEN ASSIGNED NUMBER 226.
THERE ARE NOW  9 DODG DEALERS.                      -the result


CONTINUE
```

EXHIBIT I

(continued)

```
sales of dlr 226
```

-the user wishes to know the sales of the dealer he has just added

```
SALES OF DODG DEALER NUMBER 226 ARE  337 UNITS
FOR A PENETRATION OF    .69870 PERCENT.
```

-the answer

```
CONTINUE

change in dodg sales
```

-the user realizes that some of the 337 units represent sales formerly belonging to other Dodge dealers. How did Dodge fare on the whole?

```
  9 DODG DEALERS     SALES PENETRATION
 FORMER SYSTEM       1119     2.34671
 PRESENT SYSTEM      1459     3.02305
 NET GAIN             340      .67634
```

-the net gain of 340 units comprises 9 units gained by an increase in potential and 331 units gained by the addition of a new dealer. Other Dodge dealers lost a total of 6 units to the new dealer

```
CONTINUE

report
```

-what has been the effect on each of the other makes?

```
 MAKE   --------SALES--------    --------PENETRATION--------    DEALERSHIPS   SALES PER DLR

        FORMER PRESENT CHANGE     FORMER     PRESENT    CHANGE    FMR PRS CHG    FMR PRS CHG
 FORD    8137    8164     27    17.07010   16.91545   -.15465     38  38   0    214 215   1
 CHEV   20907   21016    109    43.85713   43.54614   -.31099     47  47   0    445 447   2
 RAMB    1010    1020     10     2.11928    2.11266   -.00662      8   8   0    126 127   1
 PONT    7307    7355     48    15.32770   15.24008   -.08762     28  28   0    261 263   2
 PLYM     500     501      1     1.04939    1.03752   -.01187      5   5   0    100 100   0
 BUIC    4862    4893     31    10.19915   10.13804   -.06111     24  24   0    203 204   1
 OLDS    3828    3854     26     8.02921    7.98574   -.04347     26  26   0    147 148   1
 DODG    1119    1459    340     2.34671    3.02305    .67634      8   9   1    140 162  22
TOTAL   47670   48262    592                                     184 185   1    259 261   2

CONTINUE

quit
```

-the user terminates the program

```
RUN TERMINATED.  GOOD-BYE.
R 27.550+11.133
```

-an exit message from the program. The session has used 28 seconds of running time and 11 seconds of swap time. Cost at $10/minute comes to about $6.30

COMPUTER AIDED PREPARATION OF MAGAZINE ADVERTISEMENT FORMATS

by

Daniel S. Diamond

While most attempts toward quantification in advertising have involved budgets, media selection, and media scheduling, the present paper deals directly with magazine advertisement design. It is first proposed that before an advertisement can have any effect on the consumer, it must attract his attention. Readership is suggested as a measure of the attention-getting power of an advertisement. Next, a set of six Starch Magazine Advertisement Readership models is constructed by means of multiple regression analysis on 1,070 advertisements appearing in Life Magazine. The independent variables are predominantly advertisement format descriptors, such as size, number of colors, and position in magazine --- twelve in all. Finally, a conversational computer program is developed which requests from its user a readership objective function (a function of the six Starch readership scores for the advertisement and its cost), format restrictions, and a budget constraint. The program then prepares that advertisement format, which while conforming to all restrictions, maximizes the objective function.

1. INTRODUCTION

1.1 Attention-Getting Power and Advertisement Readership

The function of advertising is to give the consumer information about a product, to remind him of its existence, and to pursuade him to buy the product. Most would agree that "what he [the advertiser] is really concerned about is whether one advertisement or one advertising campaign will produce more sales --- or fewer sales --- than another advertisement or another campaign."(1) Before an advertisement can produce a sale, however, it must attract the consumer's attention. Only when this has been accomplished, can the content of the advertisement have any effect whatsoever on the consumer. For present purposes, it will be assumed

that the degree to which the consumer's attention is attracted toward an advertisement may be approximately measured by its Starch Magazine Advertisement Readership Scores. While the readership of an advertisement and its attention-getting power may not be identical, Starch scores present a convenient and probably fairly accurate measure of this attribute.

1.2 Starch Magazine Advertisement Readership Scores

Starch scores are the product of Daniel Starch and Staff who use a technique known as the recognition method, one of several approaches to advertisement readership.(2) Starch scores are widely used by advertising agencies and marketing personnel for evaluating advertisement performance.

> Noted-- the percent of readers who remembered that they had seen the advertisement in the particular issue.
>
> Seen-Associated--the percent of readers who have seen or read any part of the ad which clearly indicated the product or advertiser.
>
> Read Most--the percent of readers who read 50% or more of the written material in the ad.(3)

For magazines of general interest, the three Starch scores are measured for both men and women readers. For magazines like Business Week only men's scores are measured, and for magazines like Ladies' Home Journal only women's scores are measured. Starch scores are based on between 100 and 200 interviews, depending upon the publication.

For each Life Magazine readership study, interviews are conducted with 150 men and 150 women. The method by which the sampling is performed has been described by Stan M. Sargent, Senior Vice President of Daniel Starch and Staff:

> "Our manual tells the interviewers that the respondent must be 18 years of age or older. The instructions are as follows:
>
> OBTAINING A REPRESENTATIVE SAMPLE:
>
> You are assigned a specific number of interviews (quota) with men and women and we also want you to interview people of varied ages, different income levels and occupations. Interview in all types of residential areas. Do not concentrate on one neighborhood one week and another the next. Obtain a good cross-section of respondents each week."

Added Mr. Sargent, "Over a period of many issues the demographic characteristics of readers who are interviewed parallels the characteristics of the primary audience of this magazine."

1.3 Reproducibility of Starch Scores

The selection of Starch interviewees is nonprobabilisitc, and for this reason, it was doubted at one time whether the method would give reproducible results. According to Boyd and Westfall, the Advertising Research Foundation

> "replicated the methods used by the Starch organization to obtain their readership ratings. A single issue of Life was selected and a probability sample of over six hundred readers of the issue using his regular techniques. The correlation of women's noted scores on the ninety-six full-page and larger ads (measured both by ARF and Starch) was + 0.92." (4)

1.4 Elements of an Advertisement

It was pointed out earlier that it is only when an advertisement has successfully attracted the consumer's attention that its content can have any effect on him. This suggests that it is not the content of the advertisement which draws the reader, but something else. It is proposed here that it is the format of an advertisement which produces readership. Thus, an advertisement is considered to have two elements: format --- that which attracts the reader's attention, and content --- the message contained in the advertisement. More accurately, as will become clear, the word "content" will refer to the advertisement's message; "format" will have the meaning, "everything which is not content." The undertaking reported in this paper is that of developing a technique for designing advertisement formats. The selection of advertisement content, although not totally independent of format design, is not addressed here.

2. STARCH READERSHIP MODELS

2.1 Prediction of Advertisement Readership

The procedure used to design an advertisement format (actually, to select one format out of many possibilities) will depend upon the ability to predict Starch scores for an advertisement with some accurracy predominately on the basis of its proposed format. There has been some work in this area. Two examples are Twedt's article, "A Multiple Factor Analysis of Advertising Readership" (5) and Yamanak's monograph, "A Method of Prediction of Readership Score (Newspaper Ads.)" (6)

Twedt proceeded approximately as follows: First, thirty-four advertising variables (15 "mechanical" and 19 "content" --- Twedt's

terminology) were defined. Using 137 advertisements contained in an abbreviated survey issue of The American Builder (February, 1950), a readership study was conducted by the Advertising Readership Foundation. Each of the thirty-four variables was correlated independently with readership. A correlation matrix of readership score for each ad measured by the ARF study. Nineteen variables were selected as being significantly correlated with readership. A correlation matrix of readership and these nineteen variables was constructed and factor analyzed. Certain of the nineteen variables were then selected on the basis of being "factorially purest, and which also offered most promise for prediction of advertising readership." Finally, three variables --- size of advertisement (in pages), number of colors, and square inches of illustration --- all format variables --- were selected "as providing maximum prediction with minimum trouble of measurement." When readership was regressed against these three variables, the resulting correlaction coefficient was 0.76. The inclusion of nine additional variables (presumably those with the next nine highest factor loadings) produced a correlation coefficient of 0.79. Concluded Twedt, "The gain of 0.03 is obviously not worth the time involved in making these additional measurements." After the analysis of The American Builder, the three-variable regression equation was applied to one issue each of five other publications --- Automotive Industries, American Machinist, Chemical Engineering, Business Week, - and Successful Farming. The average correlation coefficient for all six issues was 0.71.

Yamanaka's contribution stems from the introduction of a technique for quantifying the qualitative aspects of an advertisement. This quantification problem is usually handled by the use of what are commonly called "dummy" variables.(7) The technique used by Yamanaka was developed by Chikio Hayashi of the Institute of Statistical Mathematics, Japanese Education Ministry.(8) It is first assumed "for simplicity" that readership can be described by a linear combination of advertisement descriptors. Given the linearity assumption, conditions for the maximization of the correlation coefficient between actual and predicted scores are derived from the point of view of the quantification of qualitative variables. Using this technique on advertisements from two Japanese morning newspapers, the Chubu-Nippon Shimbun and the Asahi-Shimbun, correlation coefficients of 0.91 and 0.89, respectively, were produced. The variables used were "space," page of appearance, position on the page, "layout," evaluation of design, evaluation of headline (both evaluations were performed by Dentsu Advertising copywriters), and past advertisement copy linage.

2.2 Variable Selection

These two examples (particularly that of Yamanaka) demonstrate that reasonably good linear predictive models of advertisement readership can be obtained. On this basis, the Starch readership score models re-

quired for this project have been constructed by means of multiple regression analysis. The source of data was Life magazine: for 1,070 of the 1,197 advertisements appearing in Life between February 7, and July 31, 1964, values for the twelve variables listed in Table 1 were measured. All advertisements appearing in this interval were used as data except: (1) those which promoted contests, special offers, etc.; (2) multiple products advertisements; and (3) advertisements for which any variable value could not be determined. Data for the dependent variable, six Starch scores for each ad, was provided by Daniel Starch and Staff.

While the readership models are intended to predict Starch scores for an advertisement on the basis of its format, two of the twelve variables are related to format only in its broad sense, "everything which is not content." One of the two, Product Class, has been chosen because of obvious wide variation in Starch scores for products in different product classes. The second, Past Advertising Expenditure, (the promotional expenditure for the product in national media during the calendar year previous to the year of advertisement publication), was selected in an attempt to have some measure of product awareness. Two additional variables, Brand and Headline Prominences, might seem to reflect content rather than format, but since they tend to indicate the approximate sizes of the brand name or logo and headline of an advertisement, they have been included. Definitions of all variables, except Number of Ads in Issue, which is simply the number of advertisements in the issue competing for the reader's attention, are presented in Tables 2 through 12.

2.3 Presentation of and Comments on Final Models

The final Starch readership score models, i.e., regression coefficients, appear with indications of statistical significance in Tables 13-25. Variable states omitted from the regression as a means of preventing the matrix singularity which would otherwise be caused by using dummy variables are indicated by dashes. Because the readership models have been constructed as a means to an end rather than an end in themselves, a discussion of them will be limited to one brief statement about the relationship of each independent variable to readership.

(1) As expected, there are marked differences in product class readership both between men and women and among different product classes. (Refer to Table 13.)

(2) The past advertising expenditure for a product does not seem to effect readership to any great extent. It is apparent, however, that the effect is nonlinear. (Refer to Table 14.)

(3) For each Starch score, the greater the number of ads in the issue, the lower the score. The effect decreases from the "noted" score

to "seen-associated" to "read most," and is greater for men than for women. (Refer to Table 15.)

(4) The contribution of advertisement size to readership decreases in the following order: double-page, "other," single-page, horizontal half-page, and vertical half-page. The score-to-score and men-to-women relationships are as in (3) above. (Refer to Table 16.)

(5) The greater the number of colors in an advertisement, the greater its readership, except in the case of the "read most" score, where color has little effect. Color effects women more than it does men. (Refer to Table 17.)

(6) The elimination of white page margins (bleed) increases readership, but very little. (Refer to Table 18.)

(7) An advertisement of one page or less will receive higher readership scores if it appears on a right-hand page rather than on a left-hand page. (Refer to Table 19.)

(8) The contribution to readership of the position of an advertisement in an issue descends in the following order: outside back cover, inside front cover, inside back cover, first, second, third, and fourth quarters. The score-to-score and men-to-women relationships are as in (3) above. (Refer to Table 20.)

(9) In general, an advertisement will receive higher readership scores if it contains a photograph rather than an illustration, either of which is better than neither. The effect is greatest in the "noted" score and smallest in the "read most" score. (Refer to Table 21.)

(10) As the number of words of text in an advertisement becomes greater than fifty, readership goes down. This is most true in the case of the "read most" score. (Refer to Table 22.)

(11) The prominence of brand identification in an advertisement seems to have little effect on readership. (Refer to Table 23.)

(12) Headline prominence has little effect on readership, except in the case of the "read most" score, where the total absence of a headline can increase text-reading significantly. (Refer to Table 24.)

2.4 Model Validation

In order to test the predictive ability of the Starch readership models, they have been used to predict Starch scores for 43 advertisements appear-

ing in the February 26, 1965, issue of Life. The set of advertisements used for validation appeared nearly eight months after the most recent advertisements used as input to the regression --- that is, they are totally independent of the original 1,070 advertisements. Since the test used here is probably the most stringent to which a predictive model can be put, the performance of the six models on the test advertisements should be a good indication of their predictive power. Figure 1 presents plots of predicted versus measured Starch scores for the set of test advertisements. The coefficients of multiple correlation and multiple determination corresponding to the plots in Figure 1 are presented in Table 26.

In brief, it seems fair to say that four of the models, all except the two for the "read most" score, are relatively good. In each of these four models, 68% or more of the variance in the Starch score has been accounted for. The two "read most" models, on the other hand, are not very good. This is probably explained by considering the meaning of the "read most" score. A person reading an advertisement will read more than half of its text only if the first half is of sufficient interest to him. What is obviously lacking is some sort of content analysis.

3. THE READERSHIP MODEL AND ADVERTISEMENT FORMAT DESIGN

3.1 Form of the Model

It has been attempted to construct a set of linear models of Starch Magazine Advertisement Readership for Life Magazine, such that one Starch score for a single advertisement appearing in Life may be predicted by the expression

$$R_n = C + \sum_i a_i x_i + \sum_j d_{j_k} y_{j_k} \pm \epsilon$$

where R_n is one of the set of six Starch readership measures and where the x's are continuous and the y's discrete independent advertisement format variables, as presented earlier. Each discrete variable has j_k states; $d_{j_k} = 1$ if the jth discrete variable takes on its kth value, $d_{j_k} = 0$, otherwise. The a_i are coefficients of the continuous variables, y_{j_k} is the contribution to R_n of the kth state of the jth discrete variable, C is a constant term, an ϵ is a small error in R_n.

Given a set of variable values and states --- which describe a single magazine advertisement format --- each model will predict one of the six Starch scores for the described advertisement, were it to appear in Life Magazine. Provided that the models are sufficiently accurate predictors of Starch scores, they can be put to good use as a tool in the preparation

of magazine advertisement formats, as will be demonstrated. Four of the models certainly qualify. The remaining two will also be used since no alternatives are currently available. As was suggested earlier, a content analysis would probably improve these two models considerably.

3.2 Advertisement Format Design Considerations

In general, the design of an advertisement format will require two types of considerations: (1) objectives for the advertisement, and (2) restrictions on the advertisement. A few possible restrictions will be pointed out. By the time the advertiser is ready to design an advertisement, he must certainly have a product. Hence, every advertisement will have at least one restriction -- it will be for a product in a fixed product class. Depending upon the product and the purpose of the advertisement, it might be desirable to present a photograph rather than an illustration, or vice versa. Similarly, the photo or illustration might be better in full color than in black and white. To illustrate, if one of the objectives of an advertisement is to emphasize the brand logo, the advertisement might require a full-color illustration. If the advertisement is intended to demonstrate a product in use, its layout might be restricted to several instructive photographs or illustrations. In addition, due to the demonstrative nature of the advertisement, the number of words of text might need to be relatively large. Other considerations might influence any of the nine advertiser-controlled variables listed in Table 1.

3.3 The Readership Objective Function

Subject to any restrictions, it would be possible to maximize, in turn, the six Starch scores for an advertisement by selecting an appropriate combination of variable states. However, it is possible that six different formats would be needed to maximize the six readership scores, and it is very likely that more than one would be necessary.

In addition, although advertisement format F_1 may be predicted to have higher Starch scores than those of format F_2, F_1 might well cost more than F_2 to insert --- the added cost of F_1 going to pay for those factors (e.g., greater number of colors, larger size, etc.) which cause its scores to be greater than those of F_2. Thus, readers-per-dollar will often be of concern to the advertiser. These considerations lead to the concept of a readership objective function, which can have either of the following forms:

$$\text{(I)}\quad V = \sum_{i=1}^{6} (W_i R_i) \qquad \text{or} \qquad \text{(II)}\quad V = \sum_{i=1}^{6} (W_i R_i)/C,$$

where W_i is a weight (or importance) assigned to the ith Starch score, R_i, and C is the cost of inserting the advertisement. The W_i are arbitrarily required to sum to one. The second form is used when readers-per-dollar is to be maximized. As an example, suppose an advertiser finds himself with a food product, the promotion for which he feels (or his research has

shown) should be aimed at women and men in the ratio of, say, 7 to 3. Also, for the particular advertisement in question, he wants as many women as possible to read the text of the ad, which contains several recipes presenting new ways to prepare the product. As far as men readers are concerned, the advertiser will be satisfied with producing brand-name remembrance. In a mathematical programming sense, then, the readership objective function for the advertisement would be $0.3R_2 + 0.7R_6$, where R_2 is the Starch "seen-associated" score for men readers, and R_6 is the "read most" score for women readers. If the advertiser wished to maximize readers-per-dollar, the objective function would become $(0.3R_2 + 0.7R_6)/C$.

3.4 Mathematical Programming Problem

The problem of selecting an advertisement format which, while subject to restrictions is to maximize a readership objective function, may be viewed as a sort of mathematical programming problem. There are, however, three essential differences between the typical mathematical programming problem and the problem encountered here. Ordinarily, one is attempting to optimize one function of two or more variables, generally subject to inequality constraints (other functions of some or all of the same variables). Here, the attempt is to optimize (maximize) a linear combination of one or more functions (one or more of the six R_n), generally subject to equality constraints on one or more of the variables (rather than functions of the variables). Unfortunately, while algorithms have been developed for some mathematical programming problems, the present solution must be found by enumeration. This is so for two reasons. First, not all combinations of variable values are legal; for example, a cover cannot be a two-page advertisement. Thus, each possible combination of variable values must be checked. Enumeration is further necessitated when the second form of objective function is being used or when a budget constraint exists: the cost of each possible format must be looked up in a table.(9)

4. THE C-A-P-M-A-F SYSTEM

4.1 Description

This approach to the preparation of magazine advertisement formats has been implemented by means of a set of conversational computer programs given the name C-A-P-M-A-F (Computer Aided Preparation of Magazine Advertisement Formats), operating under the M.I.T. Compatible Time-Sharing System. The C-A-P-M-A-F system consists of a main program, seven subroutines (all in FORTRAN), and a file containing the readership models and other necessary data. Because the system operates conversationally, all the user need know about the Time-Sharing System is the procedure for requesting program loading and execution. Once execution has begun, C-A-P-M-A-F requests all the information it needs by means

of the remote console typewriter. After each entry by the user from the console keyboard, the system checks for errors. If an entry is outside an allowable range, is illegally omitted, or if the user does not follow instructions, the requesting message is repeated. In addition, the user is given the opportunity to change an entry in the event that, although legal, it is not what he intended.

C-A-P-M-A-F has two operation modes. The first is used to test the readership models by predicting Starch scores for an advertisement which has already been published, and comparing the predicted and measured scores. Operation mode two, the design mode, requests a readership objective function and any necessary format or budget restrictions. The system then selects and displays that advertisement format which, while conforming to all restrictions, maximizes the objective function. The user may request to see up to four "runner-up" formats, and then may choose to modify the objective function, format restrictions, or budget constraint, and have C-A-P-M-A-F select the new optimal format.

4.2 Illustration

In order to illustrate the use C-A-P-M-A-F in preparing a magazine advertisement format, consider the hypothetical case of the Universal Tobacco Company, which is about to introduce a new brand: Puff Filter Cigarettes. In order to introduce Puff nationally (no promotion has appeared as yet), it has been decided to place an advertisement in Life. The format of this ad will be prepared with the assistance of C-A-P-M-A-F.

Many years of research in the cigarette market have indicated to Universal that their efforts in reaching men and women should be divided approximately in the ratio of 55 to 45. The main function of the Life advertisement would be to introduce the name "Puff." The special characteristics of Puff's "best filter yet" are to be secondary in this advertisement. It has been indicated, therefore, to split the 55 percent for men into weightings of 40 and 15 for the Starch "seen-associated" and "read most" scores, respectively, in the readership objective function. For women, the weightings were set at 35 and 10 for the same scores. It is intended to examine those formats produced with both forms of the readership objective function. Universal has no control over three of the twelve readership model variables: Puff Filter Cigarettes are in the "Tobacco and Related Products" product class; there has been no promotion; and about fifty advertisements are expected for the Life issue in question. For the remaining nine variables, in order to give C-A-P-M-A-F as much freedom as possible in maximizing the readership objective function, only those restrictions which were decided to be absolutely necessary would be imposed: The advertisement would contain two full-color photographs --- one large and one small --- the larger one showing a pleasant scene of a young couple enjoying their first Puff's, and the smaller showing the construction of the filter. The Brand Prominence would be set at its maximum level.

The C-A-P-M-A-F conversation used to aid in designing the format of the Puff advertisement is shown in Figure 2. To conserve space, part or all of several interactions have been omitted, as indicated. An outline of the conversation is as follows. The readership objective function proposed earlier has been used in its first form. The format restrictions are then entered. No budget constraint has been placed on the ad. The first modification is to change the readership objective function to its second form (readers-per-dollar). Finally, the budget limitation is set at $50,000. The formats selected by C-A-P-M-A-F are easily read from Figure 2.

5. CONCLUSION

Is the approach suggested in this paper a useful method for selecting a magazine advertising format, or is it merely of academic interest? It has been clearly shown that the degree to which an advertisement will be read can be predicted fairly accurately almost entirely on the basis of its format. The practicality of this approach depends totally on the attitude of the advertiser. If he is willing to accept advertisement readership as a measure of attention-getting power, and if he considers the maximization of attention-getting power as important to advertisement effectiveness, then he must consider the technique practical.

Table 1

Variables used in Regression Analysis

Product Class

Past Advertising Expenditure

Number of Ads in Issue

Number of Pages

Number of Colors

Bleed - No Bleed

Left or Right Page

Position in Magazine

Layout

Number of Words

Brand Prominence

Headline Prominence

Table 2

Product Class

	State
Beer, Ale, Liquor	1
Passenger Cars	2
Automotive Accessories, Gas, Oil, Tires, Trucks, Other	3
Building Materials, Paint, Wallpaper, Flooring	4
Food	5
Household Furnishings and Supplies	6
Insurance and Finance	7
Machinery, Metals, Industrials, Business Machines, Public Utilities	8
Pharmaceuticals	9
Radio, Television, Electronics, Audio Equipment	10
Tobacco and Related Products	11
Men's Clothing	12
Women's Clothing	13
Men's Toilet Goods	14
Women's Toilet Goods	15
Clothing Accessories, General Toilet Goods	16
All Others	17

Table 3

Past Advertising Expenditure(10)

Expenditure Range (X $100,000)	State
0-4	1
5-10	2
11-18	3
19-29	4
30+	5

Table 4

Number of Pages*

	State
One Page	1
Two Pages	2
One-half Page (vertical)	3
One-half Page (horizontal)	4
All Others	5

---

*Note that this variable reflects both size and orientation

Table 5

Number of Colors

	State
Black and White	1
Two Colors	2
Full Color	3

Table 6

Bleed - No Bleed**

	State
Advertisement does not use Bleed	1
Advertisement makes use of Bleed	2

**Bleed is the elimination of white page margins.

Table 7

Left or Right Page

	State
Does not apply (two-page ad, covers, etc.)	1
Half-or Full-page Ad on Left-hand Page	2
Half-or Full-page Ad on Right-hand Page	3

Table 8

Position in Magazine

	State
First Quarter	1
Second Quarter	2
Third Quarter	3
Fourth Quarter	4
Inside Front Cover	5
Inside Back Cover	6
Outside Back Cover	7

Table 9

Layout

	State
One Large Illustration	1
One Small Illustration	2
More than One Small Illustration	3
One Large Photograph	4
One Small Photograph	5
More than One Small Photograph	6
One Large Illustration and One Small Illustration	7
One Large Illustration and More than One Small Illustration	8
One Large Illustration and One Small Photo	9
One Large Illustration and More than One Small Photo	10
One Large Photo and One Small Illustration	11
One Large Photo and One Small Photo	13
One Large Photo and More than One Small Photo	14
One Small Illustration and One Small Photo	15
One Small Illustration and More than One Small Photo	16
One Small Photo and More than One Small Illustration	17
More than One Small Illustration and More than One Small Photo	18
No Photograph or Illustration	19

Table 10

Number of Words

	State
Fewer than Fifty Words	1
Fifty or More Words	2

Table 11

Brand Prominence

Brand name or sponsor is--	State
Not Present	1
Difficult to Detect	2
Easy to Miss	3
Easy to Detect	4
Almost Impossible to Miss	5

Table 12

Headline Prominence

Headline is--	State
Not Present	1
Difficult to Detect	2
Easy to Miss	3
Easy to Detect	4
Almost Impossible to Miss	5

Table 13

Product Class Coefficients

	Men Readers			Women Readers		
State†	Noted	Seen-Associated	Read Most	Noted	Seen-Associated	Read Most
1	11.5905**	12.3707**	4.7943**	-19.1961**	-19.9756**	-6.7268**
2	19.5057**	21.4411**	7.6697**	-21.1663**	-22.5102**	-6.5595**
3	11.6762**	12.6520**	6.5064**	-24.5510**	-26.6543**	-6.7979**
4	9.0203**	6.4002*	2.8277	-1.6464	-6.2085*	-0.9433
5	----	----	----	----	----	----
6	-1.2471	-0.9323	0.4598	-0.3434	-0.1651	0.6443
7	1.9910	0.9861	1.9819**	-16.5995	-19.0118**	-4.6676**
8	3.9241**	3.6092**	2.8454**	-10.4453**	-12.8830**	-2.5963**
9	3.0698*	2.2312	1.5013*	-4.7141**	-5.8439**	-1.4719
10	8.3489**	8.5984**	3.5805**	-8.5532**	-9.5103**	-2.7763**
11	5.7084**	6.3266**	0.2650	-18.8901**	-19.4298**	-7.1611**
12	14.7261**	14.5064**	6.2608**	-9.7710**	-10.8504**	-1.3181
13	-5.6468*	-11.4612**	-1.1344	10.3582**	9.3084**	6.4042**
14	12.3581**	12.9834**	6.1198**	-16.4946**	-17.2523**	-3.6097**
15	-12.8196**	-17.8393**	-2.9818**	0.3378	-1.6648	1.7306*
16	4.5563**	4.7286**	1.9580*	-5.1551**	-6.3054**	-2.3381**
17	6.7455**	6.6168**	4.1819**	-9.8776**	-10.8927**	-1.4170*

† Refer to Table 2

* Significant at the 5% level

**Significant at the 1% level

Table 14

Past Advertising Expenditure Coefficients

	Men Readers			Women Readers		
State[†]	Noted	Seen-Associated	Read Most	Noted	Seen-Associated	Read Most
1	----	----	----	----	----	----
2	-0.9371	-0.5519	-1.3266**	2.7711**	2.8644**	0.6504
3	0.8117	1.5566	-0.9377*	-0.3117	-0.0667	-1.1339*
4	1.9080*	2.5815**	-0.5631	2.1297*	2.6874**	-0.3627
5	2.7438**	3.3250**	0.3980	5.6850**	6.5165**	0.7878

[†] Refer to Table 3

* Significant at the 5% level

**Significant at the 1% level

Table 15

Number of Ads in Issue Coefficients

Men Readers			Women Readers		
Noted	Seen-Associated	Read Most	Noted	Seen-Associated	Read Most
-0.1395**	-0.1233**	-0.0802**	-0.1056**	-0.0959**	-0.0524**

**Significant at the 1% level

Table 16

Number of Pages Coefficients

	Men Readers			Women Readers		
State†	Noted	Seen-Associated	Read Most	Noted	Seen-Associated	Read Most
1	----	----	----	----	----	----
2	16.7655**	15.1207**	4.0288**	12.9481**	12.7032**	1.8483
3	-6.8914**	-5.8631**	-1.3969**	-9.4308**	-8.6512**	-2.3281**
4	-6.5803**	-5.8516**	-0.5948	-9,1647**	-8.5328**	-2.3898*
5	8.4876**	6.9813**	2.2357	8.7862**	8.9503**	2.6052

† Refer to Table 4

* Significant at the 5% level

** Significant at the 1% level

Table 17

Number of Colors Coefficients

	Men Readers			Women Readers		
State†	Noted	Seen-Associated	Read Most	Noted	Seen-Associated	Read Most
1	-7.1856**	-6.1041**	-0.5534	-11.9463**	-10.3119**	-1.9030**
2	-5.2627**	-3.5459**	-0,5073	-7.6150**	-6.4615**	-2.1109**
3	----	----	----	----	----	----

† Refer to Table 5

** Significant at the 1% level

Table 18

Bleed -- No Bleed Coefficients

	Men Readers			Women Readers		
State†	Noted	Seen-Associated	Read Most	Noted	Seen-Associated	Read Most
1	----	----	----	----	----	----
2	1.1661*	0.4975	0.9522**	2.3106**	1.9864**	1.2123**

† Refer to Table 6

* Significant at the 5% level

** Significant at the 1% level

Table 19

Left or Right Page Coefficients

	Men Readers			Women Readers		
State†	Noted	Seen-Associated	Read Most	Noted	Seen-Associated	Read Most
1	-7.1476**	-6.6993**	-2.5641	-4.1458	-4.9886*	-0.3173
2	-3.1159**	-2.7924**	-0.2965	-3.5151**	-3.0433**	-0.1665
3	----	----	----	----	----	----

† Refer to Table 7

* Significant at the 5% level

** Significant at the 1% level

Table 20

Position in Magazine Coefficients

State†	Men Readers: Noted	Men Readers: Seen-Associated	Men Readers: Read Most	Women Readers: Noted	Women Readers: Seen-Associated	Women Readers: Read Most
1	----	----	----	----	----	----
2	-1.4177*	-1.2969	-0.5980	-0.7769	-0.2797	-0.0306
3	-2.6807**	-2.3602**	-0.2671	-2.2854**	-1.6745*	0.0661
4	-2.9507**	-2.2841**	-0.5530	-2.7811**	-2.1355**	-0.4622
5	11.5497**	9.0402**	0.9295	12.6888**	11.5751**	1.3010
6	3.7147	3.2280	1.5280	0.5015	-0.1561	1.0368
7	30.5751**	29.7921**	3.7246*	21.6501**	21.6207**	-1.3748

† Refer to Table 8

* Significant at the 5% level

** Significant at the 1% level

Table 21

Layout Coefficients

	Men Readers			Women Readers		
State†	Noted	Seen-Associated	Read Most	Noted	Seen-Associated	Read Most
1	-2.1009	-1.1604	-0.9792	-3.9270	-4.1479*	-1.1800
2	-5.5272**	-4.0761*	-0.6212	-6.9972**	-5.1666*	-1.7612
3	-5.7626**	-4.8930**	-0.7962	-6.1268**	-4.6741**	0.4415
4	----	----	----	----	----	----
5	-4.5887**	-4.1552**	0.2086	-2.8756	-1.9287	2.7534**
6	-4.5480**	-4.1097**	-1.4616**	-1.6980	-0.6418	0.3764
7	-2.4658	-2.7533	-1.8139	-5.6807*	-4.7685*	-1.3387
8	-8.3204**	-8.4159**	-2.6524	-1.4361	-0.8015	-0.6603
9	3.7575	4.3385	-4.3058*	-9.2201*	-8.7606**	-4.7040*
10	-7.6353*	-5.0363	-4.7727**	-7.3384	-4.9051	-1.2240
11	-0.6046	-0.9141	-0.8969	-2.1214	-1.9517	-1.4742
12	-2.8576	-2.4345	-0.7553	-3.3222	-2.6729	-1.9749*
13	-2.0430**	-2.0848**	-2.3419**	-0.4754	-0.1307	-1.4686**
14	-3.0070**	-3.1840**	-1.5806**	-0.0063	0.5124	-0.3627
15	-0.7949	0.7746	-1.3216	-2.7569	-1.2586	-0.7010
16	-7.5820**	-7.0439**	-0.8631	-8.3938**	-7.3309**	-1.9160
17	-3.1132	-2.0694	-0.6427	-11.4373**	-10.7137**	-2.1212
18	-3.5014	-4.0371	-2.6726	-0.9048	0.6199	-0.9712
19	-10.5719**	-7.7140**	0.1231	-11.5402**	-7.8976**	-0.7260

† Refer to Table 9

* Significant at the 5% level

** Significant at the 1% level

Table 22

Number of Words Coefficients

State†	Men Readers Noted	Men Readers Seen-Associated	Men Readers Read Most	Women Readers Noted	Women Readers Seen-Associated	Women Readers Read Most
1	1.9764*	1.7760 *	3.9123**	4.3260**	4.8185**	5.3000**
2	----	----	----	----	----	----

† Refer to Table 10

* Significant at the 5% level

** Significant at the 1% level

Table 23

Brand Prominence Coefficients

State†	Men Readers Noted	Men Readers Seen-Associated	Men Readers Read Most	Women Readers Noted	Women Readers Seen-Associated	Women Readers Read Most
1	-0.9560	-1.3820	2.0120	0.7828	0.7944	1.4216
2	-----	----	----	----	----	----
3	-0.0637	0.8281	-0.9069**	-0.2548	0.5085	-0.8026*
4	-0.4582	1.0392	-0.8194	-1.1429	0.2711	-0.2905
5	-0.0156	1.6474	-1.0828*	0.1647	1.8548	-0 4859

† Refer to Table 11

* Significant at the 5% level

** Significant at the 1% level

Table 24

Headline Prominence Coefficients

	Men Readers			Women Readers		
State†	Noted	Seen-Associated	Read Most	Noted	Seen-Associated	Read Most
1	-0.1317	-0.8529	3.4003**	1.4736	0.0716	4.5826**
2	0.6950	0.6491	0.1897	1.1429	1.0075	0.6797
3	1.1217	1.3221*	-0.2419	0.1730	0.0027	-0.0664
4	----	----	----	----	----	----
5	-0.6936	-0.4270	0.3745	-1.3232	-1.0931	0.0383

† Refer to Table 12

* Significant at the 5% level

** Significant at the 1% level

Table 25

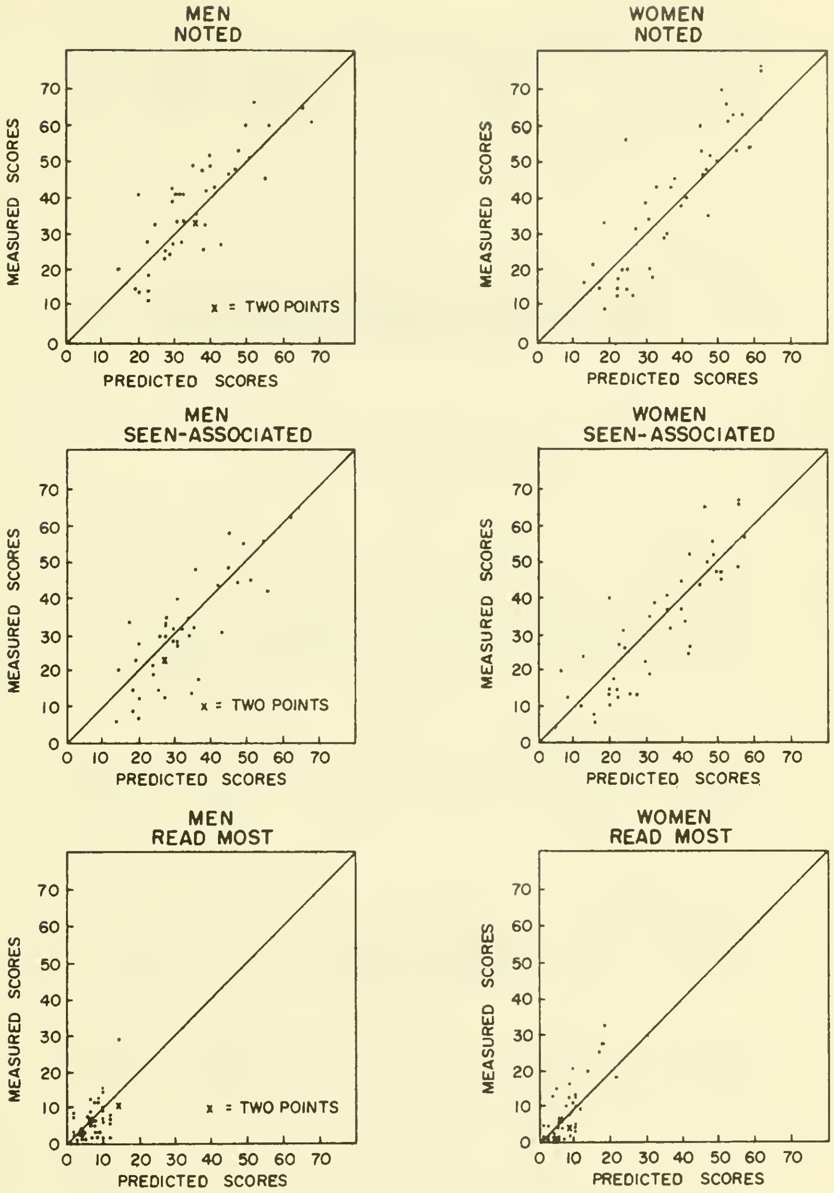
Regression Equation Constant Terms

Men Readers			Women Readers		
Noted	Seen-Associated	Read Most	Noted	Seen-Associated	Read Most
42.0259	35.5477	9.6826	57.9110	51.8479	12.7425

Table 26

Validation of Readership Models -- Coefficients of Multiple Correlation (R) and Multiple Determination (R^2)

	Men Readers			Women Readers		
	Noted	Seen-Associated	Read Most	Noted	Seen-Associated	Read Most
R	0.8240	0.8297	0.5643	0.8581	0.8588	0.7507
R^2	0.6790	0.6884	0.3185	0.7363	0.7376	0.5636


MEN NOTED

WOMEN NOTED

MEN SEEN-ASSOCIATED

WOMEN SEEN-ASSOCIATED

MEN READ MOST

WOMEN READ MOST

Figure 1. Validation of readership models.

```
                  ENTRY TO C-A-P-M-A-F

ENTER AN IDENTIFYING PRODUCT NAME, RUN NUMBER, AND DATE

X----PRODUCT NAME----X          NUM          XX/YY/ZZ
puff filter cigarettes          001           1/27/66




INDICATE WHETHER THE SYSTEM IS TO BE USED IN MODE 1 OR
MODE 2 BY ENTERING, UNDER THE X AT THE LEFT-HAND MARGIN,
X     THE NUMBER OF THE OPERATION MODE DESIRED.
2


                 C-A-P-M-A-F - MODE TWO


PUFF FILTER CIGARETTES          001           1/27/66

DEFINE AN ADVERTISEMENT READERSHIP OBJECTIVE FUNCTION--

1)  ENTER WEIGHTINGS (WHICH MUST SUM TO EXACTLY 100.0)
    UNDER ONE OR MORE OF THE READERSHIP SCORES TO BE
    LISTED BELOW.

       MEN READERS                 WOMEN READERS

NOTED  SEEN-ASSOC  RD MOST * NOTED  SEEN-ASSOC  RD MOST
 XX.       XX.       XX.      XX.       XX.       XX.
           40        15                 35        10



    2)  INDICATE THAT 'READERS PER DOLLAR' IS TO BE MAXI-
    MIZED BY ENTERING A '1' UNDER THE 'X' AT THE LEFT.
    X               OTHERWISE, RETURN.
    1
```

Figure 2. The C-A-P-M-A-F conversation used to aid in designing the format of the Puff advertisement.

```
A READERSHIP OBJECTIVE FUNCTION FOR THE ADVERTISEMENT
UNDER CONSIDERATION HAS BEEN DEFINED AS FOLLOWS---

                 .40 (SEEN ASSOC,MEN)
                          +
                 .15 (READ MOST,MEN)
                          +
                 .35 (SEEN-ASSOC,WOMEN)
                          +
                 .10 (READ MOST,WOMEN)


---THE SUM TO BE DIVIDED BY THE COST OF THE AD---


X     TO MODIFY, ENTER A '1' UNDER THE X AT LEFT

                 ADVERTISEMENT CONSTRAINTS

PLEASE ENTER CONSTRAINT VALUES UNDER X'S AS INDICATED.

(FOR PROPER CODING, SEE THE C-A-P-M-A-F USER'S MANUAL.)

                         CONSTRAINT
VAL.   NUMBER                      NAME

XX        1      PRODUCT CLASS
11

X     TO MODIFY, ENTER '1'.  OTHERWISE, RETURN.


XX        2      PAST ADVERTISING EXPENDITURE
 1

X     TO MODIFY, ENTER '1'.  OTHERWISE, RETURN.


XX        3      NUMBER OF ADS IN ISSUE
50

X     TO MODIFY, ENTER '1'.  OTHERWISE, RETURN.
```

Figure 2. Continued.

```
XX        4     NUMBER OF PAGES

     CONSTRAINT NUMBER  4 HAS BEEN UNSPECIFIED.
X    TO MODIFY, ENTER '1'.  OTHERWISE, RETURN.

XX        5     NUMBER OF COLORS
 3

X    TO MODIFY, ENTER '1'.  OTHERWISE, RETURN.

XX        6     BLEED - NO BLEED

     CONSTRAINT NUMBER  6 HAS BEEN UNSPECIFIED.
X    TO MODIFY, ENTER '1'.  OTHERWISE, RETURN.

XX        7     LEFT OR RIGHT PAGE

     CONSTRAINT NUMBER  7 HAS BEEN UNSPECIFIED.
X    TO MODIFY, ENTER '1'.  OTHERWISE, RETURN.

XX        8     POSITION IN MAGAZINE

     CONSTRAINT NUMBER  8 HAS BEEN UNSPECIFIED.
X    TO MODIFY, ENTER '1'.  OTHERWISE, RETURN.

XX        9     LAYOUT
13

X    TO MODIFY, ENTER '1'.  OTHERWISE, RETURN.

XX       10     NUMBER OF WORDS

     CONSTRAINT NUMBER 10 HAS BEEN UNSPECIFIED.
X    TO MODIFY, ENTER '1'.  OTHERWISE, RETURN.
```

Figure 2. Continued.

```
XX       11     BRAND PROMINENCE
5


X    TO MODIFY, ENTER '1'.  OTHERWISE, RETURN.
1


XX       11     BRAND PROMINENCE
 5


X    TO MODIFY, ENTER '1'.  OTHERWISE, RETURN.


XX       12     HEADLINE PROMINENCE


     CONSTRAINT NUMBER 12 HAS BEEN UNSPECIFIED.

X    TO MODIFY, ENTER '1'.  OTHERWISE, RETURN.

               BUDGET CONSIDERATION


ENTER THE MAXIMUM ALLOWABLE COST, IF ANY, OF THE AD
UNDER CONSIDERATION. THE LEAST EXPENSIVE AD IN LIFE
$XXXXXX.               COSTS $19050.




GIVEN THIS SET OF CONSTRAINTS, C-A-P-M-A-F WILL EXAMINE
  APPROXIMATELY   2100 DIFFERENT ADVERTISEMENT FORMATS.

               FINAL C-A-P-M-A-F SELECTION


SUBJECT TO THE CONSTRAINTS ENTERED EARLIER, C-A-P-M-A-F
HAS CHOSEN THE ADVERTISEMENT FORMAT DISPLAYED BELOW AS
MAXIMIZING THE DEFINED READERSHIP OBJECTIVE FUNCTION.

PRODUCT CLASS                     TOBACCO + RELATED PRODS

PAST ADVERTISING EXPENDITURE                UP TO $ 449K

NUMBER OF ADS IN ISSUE                                50

NUMBER OF PAGES                     ONE-HALF PAGE (VERT)*

NUMBER OF COLORS                              FULL COLOR

BLEED - NO BLEED                                NO BLEED*
```

Figure 2. Continued.

```
LEFT OR RIGHT PAGE                                        RIGHT*
POSITION IN MAGAZINE                             FIRST QUARTER*
LAYOUT                                  1 LG PHOTO + 1 SM PHOTO
NUMBER OF WORDS                                  FEWER THAN 50*
BRAND PROMINENCE                                      VERY HIGH
HEADLINE PROMINENCE                                        LOW*

                    * SELECTED BY C-A-P-M-A-F

      C-A-P-M-A-F HAS PREDICTED THE FOLLOWING READERSHIP
                    SCORES FOR THIS AD.

        MEN READERS                         WOMEN READERS

NOTED  SEEN-ASSOC   RD MOST * NOTED  SEEN-ASSOC   RD MOST
 34       32           5       29       27           5



THE COST OF THIS AD IN LIFE WOULD BE APPROX.    $ 31660
THE OBJECTIVE FUNCTION WAS MAXIMIZED AT          .000735



INDICATE, UNDER THE 'X' BELOW, THE NUMBER, IF ANY, OF
          'RUNNER-UP' FORMATS TO BE DISPLAYED.
X                   OTHERWISE, RETURN.
1

                    RUNNER-UP NUMBER 1
```

(Only changed format element shown)

```
HEADLINE PROMINENCE                                  VERY LOW*

      C-A-P-M-A-F HAS PREDICTED THE FOLLOWING READERSHIP
                    SCORES FOR THIS AD.

        MEN READERS                         WOMEN READERS

NOTED  SEEN-ASSOC   RD MOST * NOTED  SEEN-ASSOC   RD MOST
 34       30           8       30       26           9
```

Figure 2. Continued.

```
THE COST OF THIS AD IN LIFE WOULD BE APPROX.    $ 31660

THE OBJECTIVE FUNCTION WAS EVALUATED AT          .000733



        TO MODIFY                                      ENTER

 OBJECTIVE FUNCTION ONLY                                 1
    RESTRICTIONS ONLY                                    2
BUDGET CONSIDERATION ONLY                                3

X                  OTHERWISE, RETURN.
1
```

(Interactions Omitted)

```
A READERSHIP OBJECTIVE FUNCTION FOR THE ADVERTISEMENT
UNDER CONSIDERATION HAS BEEN DEFINED AS FOLLOWS---

                .40 (SEEN ASSOC,MEN)

                         +

                .15 (READ MOST,MEN)

                         +

                .35 (SEEN-ASSOC,WOMEN)

                         +

                .10 (READ MOST,WOMEN)


X     TO MODIFY, ENTER A '1' UNDER THE X AT LEFT


               FINAL C-A-P-M-A-F SELECTION


SUBJECT TO THE CONSTRAINTS ENTERED EARLIER, C-A-P-M-A-F
HAS CHOSEN THE ADVERTISEMENT FORMAT DISPLAYED BELOW AS
MAXIMIZING THE DEFINED READERSHIP OBJECTIVE FUNCTION.

PRODUCT CLASS                     TOBACCO + RELATED PRODS

PAST ADVERTISING EXPENDITURE                 UP TO $449K
```

Figure 2. Continued.

```
NUMBER OF ADS IN ISSUE                                   50

NUMBER OF PAGES                                   ONE PAGE*

NUMBER OF COLORS                                 FULL COLOR

BLEED - NO BLEED                                     BLEED*

LEFT OR RIGHT PAGE                          DOES NOT APPLY*

POSITION IN MAGAZINE                     OUTSIDE BACK COVER*

LAYOUT                                1 LG PHOTO + 1 SM PHOTO

NUMBER OF WORDS                              FEWER THAN 50*

BRAND PROMINENCE                                  VERY HIGH

HEADLINE PROMINENCE                                    LOW*


                    * SELECTED BY C-A-P-M-A-F


    C-A-P-M-A-F HAS PREDICTED THE FOLLOWING READERSHIP
                     SCORES FOR THIS AD.


          MEN READERS                     WOMEN READERS

NOTED   SEEN-ASSOC   RD MOST * NOTED   SEEN-ASSOC   RD MOST
 66         61           9       59        54          7




THE COST OF THIS AD IN LIFE WOULD BE APPROX.    $ 79350

THE OBJECTIVE FUNCTION WAS MAXIMIZED AT         45.301834




INDICATE, UNDER THE 'X' BELOW, THE NUMBER, IF ANY, OF
         'RUNNER-UP' FORMATS TO BE DISPLAYED.
X                   OTHERWISE, RETURN.
1


                     RUNNER-UP NUMBER 1
```

(Only changed format element shown)

```
HEADLINE PROMINENCE                               VERY LOW*
```

Figure 2. Continued.

```
      C-A-P-M-A-F HAS PREDICTED THE FOLLOWING READERSHIP
                       SCORES FOR THIS AD.

          MEN READERS                      WOMEN READERS

NOTED   SEEN-ASSOC   RD MOST * NOTED   SEEN-ASSOC   RD MOST
 65         60          12       59        53          10



THE COST OF THIS AD IN LIFE WOULD BE APPROX.     $ 79350

THE OBJECTIVE FUNCTION WAS EVALUATED AT        45.245349



           TO MODIFY                                    ENTER

 OBJECTIVE FUNCTION ONLY                                  1
    RESTRICTIONS ONLY                                     2
BUDGET CONSIDERATION ONLY                                 3

X                  OTHERWISE, RETURN.
3


                   BUDGET CONSIDERATION


ENTER THE MAXIMUM ALLOWABLE COST, IF ANY, OF THE AD
UNDER CONSIDERATION. THE LEAST EXPENSIVE AD IN LIFE
$XXXXXX.                COSTS $19050.
  50000


                   FINAL C-A-P-M-A-F SELECTION


SUBJECT TO THE CONSTRAINTS ENTERED EARLIER, C-A-P-M-A-F
HAS CHOSEN THE ADVERTISEMENT FORMAT DISPLAYED BELOW AS
MAXIMIZING THE DEFINED READERSHIP OBJECTIVE FUNCTION.

PRODUCT CLASS                        TOBACCO + RELATED PRODS

PAST ADVERTISING EXPENDITURE                   UP TO $449K

NUMBER OF ADS IN ISSUE                                  50

NUMBER OF PAGES                        ONE-HALF PAGE (HORIZ)*
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Figure 2. Continued.

```
NUMBER OF COLORS                                    FULL COLOR

BLEED - NO BLEED                                        BLEED*

LEFT OR RIGHT PAGE                                 .    RIGHT*

POSITION IN MAGAZINE                            FIRST QUARTER*

LAYOUT                                1 LG PHOTO + 1 SM PHOTO

NUMBER OF WORDS                                 FEWER THAN 50*

BRAND PROMINENCE                                     VERY HIGH

HEADLINE PROMINENCE                                       LOW*


                   * SELECTED BY C-A-P-M-A-F


   C-A-P-M-A-F HAS PREDICTED THE FOLLOWING READERSHIP
                     SCORES FOR THIS AD.


      MEN READERS                    WOMEN READERS

NOTED   SEEN-ASSOC   RD MOST * NOTED  SEEN-ASSOC   RD MOST
 36         32          7       32        29          6




THE COST OF THIS AD IN LIFE WOULD BE APPROX.     $ 37185

THE OBJECTIVE FUNCTION WAS MAXIMIZED AT         24.583295




INDICATE, UNDER THE 'X' BELOW, THE NUMBER, IF ANY, OF
        'RUNNER-UP' FORMATS TO BE DISPLAYED.
X                OTHERWISE, RETURN.
1


                    RUNNER-UP NUMBER 1
```

(Only changed format element shown)

```
HEADLINE PROMINENCE                                  VERY LOW*
```

Figure 2. Continued.

```
      C-A-P-M-A-F HAS PREDICTED THE FOLLOWING READERSHIP
                      SCORES FOR THIS AD.

          MEN READERS                         WOMEN READERS

NOTED  SEEN-ASSOC  RD MOST * NOTED  SEEN-ASSOC  RD MOST
 35        31         10      32        28         10



THE COST OF THIS AD IN LIFE WOULD BE APPROX.    $ 37185

THE OBJECTIVE FUNCTION WAS EVALUATED AT        24.526810



          TO MODIFY                                ENTER

  OBJECTIVE FUNCTION ONLY                            1
     RESTRICTIONS ONLY                               2
BUDGET CONSIDERATION ONLY                            3

X                   OTHERWISE, RETURN.



X    TO RE-ENTER C-A-P-M-A-F, ENTER A '1' UNDER THE X AT
     THE LEFT.  OTHERWISE, RETURN.


                   EXIT FROM C-A-P-M-A-F
```

Figure 2. Continued.

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A Model of Adaptive Control of Promotional Spending

122-65

John D. C. Little

Working Paper

Revised July 1, 1965

Abstract

A simple model of adaptive control of promotional spending is analyzed. In the model, company sales (and therefore profits) are functions of promotional spending rate. Sales response to the promotion changes with time as a result of changes in a parameter of the sales response function. Information about sales response is collected in each time period by performing an experiment. On the basis of present and past information, the promotion rate is set to maximize expected profit in the next period. The experiment is chosen to minimize the combined costs of imperfect information and experimentation.

A numerical example is studied analytically and by simulation. The adaptive system appears to work well. In a sensitivity analysis, the system based on one underlying model of the market is found to perform well when the underlying model is changed in various ways.

A Model of Adaptive Control of Promotional Spending

John D. C. Little*

1. Introduction

A company must assemble marketing information, use it to modify its conception of the market, use the revised conception to make marketing decisions, and then arrange for the collection of new information. In short, a company needs a control system for its marketing variables.

Obviously every company has some procedure for determining its marketing actions, but usually the relationships between data inputs and decision outputs are not at all formally specified. Our interest is in studying possible inputs and possible relationships to determine their effect on overall company performance. We are particularly interested is systems that are designed to handle changing market conditions.

Formal systems design in the sense we seek will take considerable empirical and theoretical development. Marketing variables are many and so are the possible sources and forms of information. However, we can at least start the job by investigating a simple marketing system that involves some of the important ideas.

The system consists of a model of the marketing process to be controlled, a means of using the model to set values for the marketing

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* Sloan School of Management, MIT, Cambridge, Massachusetts, 02139. Work reported was supported in part by Project MAC, an MIT research program sponsored by the Advanced Research Projects Agency, Department of Defense, under the Office of Naval Research Contract No. Nonr-4102(01). Reproduction in whole or in part is permitted for any purpose of the United States Government.

variables, and a measurement device for keeping the model up to date. The model of the process is briefly as follows: company sales (and therefore profits) are functions of a single variable, a rate of spending money on promotion. The sales response to promotion changes with time. The change is the result of a single changing parameter in the sales response function.

The control cycle operates as shown in Figure 1: Based on the sales model, a calculation is made that sets the promotion rate to maximize expected profit in the next time period. An experiment is then designed to monitor the effectiveness of the promotion. The results are then implemented, the market responds, and some sales rate is produced. The sales data thereby generated represents new information, which is then combined with old information to update the sales model. The cycle is then repeated.

In designing the control system, we must specify how to combine new and old information and we must determine what size of experiment to perform. We plan to combine new and old information so as to maximize the expected profit from the decisions that use the information. The size of the experiment will be chosen to minimize the sum of the losses arising from imperfect information about sales response and from the cost of performing the experiment.

Thus, the problem is set up according to the principles of statistical decision theory. The problem has a complication not found in elementary applications, however, because sales response is changing with time, and, as a result, the value of a piece of information deteriorates with its age.

For a measuring instrument, we shall work with direct sales experiments. That is, we shall take different groups of customers,

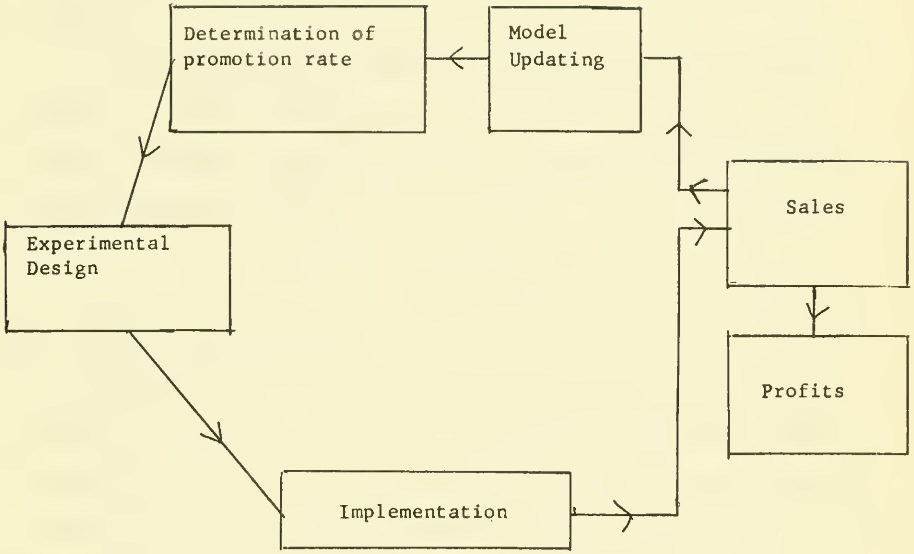

Figure 1. Cycle of adaptive control of promotion.

say, people in different geographical areas, give them different experimental treatments, and observe the effect on sales. Other, less direct devices for monitoring sales response are of course possible.

The overall system is certainly an idealization of real world operations, but it is perhaps not too far from practicality. At least one company we know goes around the feedback loop of Figure 1 in an informal way now.

2. Sales and Profit Models

The profit model expresses company profit rate in terms of its sales rate, promotion rate, fixed cost rate, and incremental profit on sales, the latter taken to be constant. To make the quantities as concrete as possible we shall give them specific units. In particular, money flows will be expressed in dollars per household per year (dol./hh.yr.) Let

s = sales rate. (dol./hh.yr.)
x = promotion rate. "
p = profit rate. "
c = fixed cost rate. "
m = gross margin, the incremental profit as a fraction of sales. (dimensionless)

The model of company profit is

$$p = ms - x - c \quad (2.1)$$

Notice that promotion enters here as a fixed cost. Thus we are not planning to consider variable cost promotional activities (e.g. price-off deals), although there is no conceptual difficulty in so doing.

The sales model is set up as a simple quadratic in promotion rate. We suppose that, for a given fixed time period, sales response has the general shape shown in Figure 2 and that the curve can be approximated, at least near the current operating point, by a quadratic function of x:

$$(2.2) \qquad s = \alpha + \beta x - \gamma x^2 .$$

The parameters α, β, and γ are constants for the fixed time period (they may be thought of as giving the average sales rate over the period) but some of them may vary from period to period.

The value of x, say x^*, that maximizes profit is easily found to be

$$(2.3) \qquad x^* = (m\beta - 1)/2m\gamma .$$

If the company uses x instead of x^*, the loss rate, ℓ, relative to maximum profit is

$$\ell(x) = p(x^*) - p(x) ,$$

which, using (2.1), (2.2), and (2.3), becomes

$$(2.4) \qquad \ell(x) = m\gamma(x - x^*)^2 .$$

3. Changes in Sales Response

If the sales response parameters α, β, and γ were known, we would set promotion rate to x^* and obtain the loss $\ell(x^*) = 0$. However, the parameters are presumably fairly difficult to measure and we ordinarily expect to come up with some non-optimal x and therefore to incur a relative loss.

If the parameters were constant over time, we would put a big effort into measuring them right away, because the extra profit from increased accuracy would extend far into the future. However, it is difficult to believe that in practice the parameters stay constant.

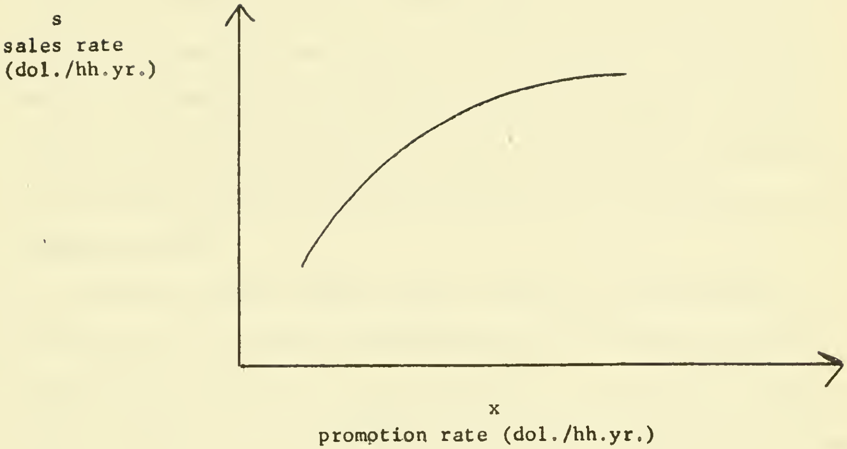

Figure 2. Shape of sales response to promotion in a fixed time period.

For example, competitive activity, product changes, changes in the quality of the promotion, and shifts in economic conditions lead us to expect shifts in response. Consequently, an expensive effort to learn the parameters immediately cannot be justified. On the other hand, the parameters may change fairly slowly with time, in which case some effort is worthwhile. In each time period new information is collected, combined with the old and used to set operations in the immediate future.

To build a fairly simple model of changing sales response, we shall suppose that α and β change with time in a specified way involving some randomness, but that γ does not. As a matter of notation, we shall use the tilde ($\sim$) when we wish to emphasize that some quantity is being viewed as a random variable. Furthermore, α, β, s and x usually depend on t, and when this needs emphasis we shall write $\alpha(t)$, $\beta(t)$, etc.

At a fixed time period, t, we assume that national sales rate for the product is

$$\tilde{s} = \tilde{\alpha} + \tilde{\beta}x - \gamma x^2 \quad \text{(dol./hh.yr.)} \tag{3.1}$$

and that

$\tilde{\alpha}(t)$ has relatively high variance from time period to time period

$\tilde{\beta}(t)$ is dependent on $\tilde{\beta}(t-1)$,

$\tilde{\alpha}$ and $\tilde{\beta}$ are independent,

γ is a known constant.

As implied by (2.3) and as will be seen, the current information about α does not directly affect the optimal x. However, information about α may make it possible to learn more about β in an experiment. The assumption of high variance for α, (high conditional

variance given the previous α's if successive α's are considered dependent) may be fairly realistic and, in any case, simplifies the statistical analysis by removing α as a contributor to information about β.

The assumption that γ is known and constant seems quite unrealistic, but we shall argue later that this may not be too serious to the successful operation of the system.

The response parameter β will be considered to be generated by a random walk. One possibility is $\tilde{\beta}(t) = \tilde{\beta}(t-1) + \tilde{\epsilon}_\beta(t)$, where

$$\tilde{\epsilon}_\beta(t) = \text{a random variable with mean} = 0 \text{ and variance} = \sigma_\beta^2 .$$

We shall take $\tilde{\epsilon}_\beta(t)$ to be normally distributed and independent of previous values of β and ϵ_β. The difficulty with the above random walk is that β is likely to wander unrealistically far from its starting value. Therefore, we shall hypothesize a long run average value and a tendency for β to return to that value. Specifically, let

β^o = the long run average value of $\tilde{\beta}(t)$.

k = a constant, $0 \leq k \leq 1$.

We take as our model of changing β:

$$\tilde{\beta}(t) = k\,\tilde{\beta}(t-1) + (1-k)\beta^o + \tilde{\epsilon}_\beta(t) \tag{3.2}$$

As $k \rightarrow 1$, $\beta(t)$ is increasingly dependent on $\beta(t-1)$ and also wanders more and more freely from β^o . As $k \rightarrow 0$, β becomes independent from time period to time period. Figure 3 shows a possible sequence of $\beta(t)$ versus t. To make $\beta(t)$ more operationally meaningful, we observe that if the promotion rate is held constant, the sales rate consists of a constant plus a term directly proportional to $\beta(t)$.

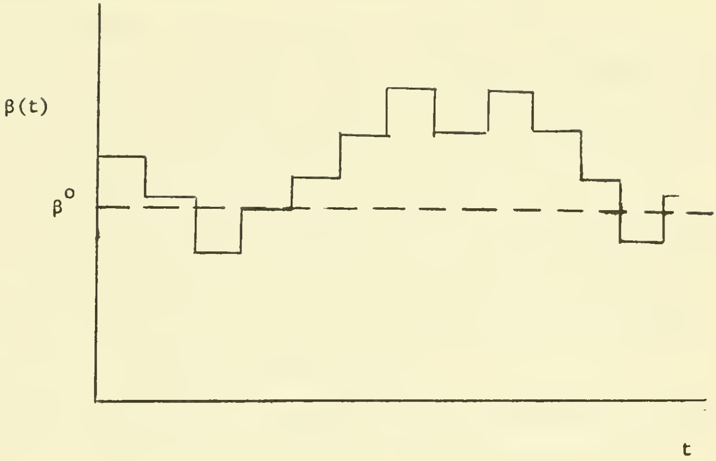

Figure 3. Sketch of a possible variation of the sales response parameter, β, with time period, t.

4. An Experiment to Measure Current Sales Response

In the previous two sections we have described the environment in which the company is operating. Next we describe the control system. Our starting place in the cycle of Figure 1 will be the point of designing an experiment to measure sales response.

Measuring sales response essentially means measuring β, since α does not enter decisions and γ is taken to be a known constant. Information about β will be collected by operating different groups of market areas at different promotion rates.

Although the national average sales rate is given by (3.1) we suppose that individual markets differ in sales rate because of local random variations. For some fixed t suppose that the national parameters take on the specific values $\tilde{\alpha} = \alpha$ and $\tilde{\beta} = \beta$. We assume sales in a market are then given by

$$(4.1) \qquad \tilde{s} = s(x) + \tilde{\epsilon}$$

where $\tilde{s}$ = sales rate in the market. (dol./hh.yr.)

$s(x) = \alpha + \beta x - \gamma x^2$ = national sales rate. (dol./hh.yr.)

$\tilde{\epsilon}$ = a random variable for the market. (dol./hh.yr.)

We assume that $\tilde{\epsilon}$ is normally distributed, is independent from market to market, and has mean = 0 and variance = σ^2.

The experiment is sketched in Figure 4. Suppose that at t we have picked a promotion rate, $x_o(t)$. This will be used everywhere except that in n markets we shall use a deliberately low value, x_1, and in another n a deliberately high value, x_2. We take

$$(4.2) \qquad x_1 = x_o(t) - \Delta/2$$

$$x_2 = x_o(t) + \Delta/2$$

where Δ is a design constant yet to be selected.

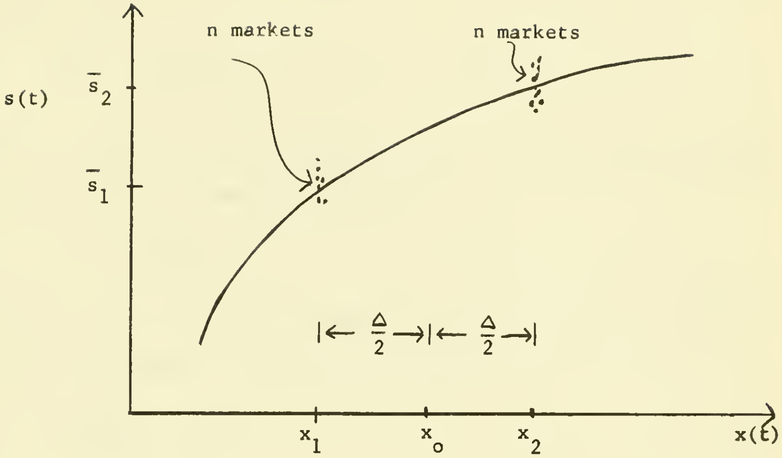

Figure 4. The sales experiment. A group of n markets is given a promotion rate $\Delta/2$ greater than the national average, x_o, and another group of n is given a rate $\Delta/2$ less.

Let $\bar{s}_1$ and $\bar{s}_2$ be the observed mean sales rates in the groups of markets x_1 and x_2 respectively. An estimate of $\beta(t)$ can be computed from the experimental data:

$$(4.3) \qquad \hat{\beta}(t) = (1/\Delta)(\bar{s}_2 + \gamma x_2^{2} - \bar{s}_1 - \gamma x_1^{2}) .$$

This will be called the "experimental mean". It is a random variable because $\bar{s}_1$ and $\bar{s}_2$ are random variables. Let*

$$v = V\left[\hat{\beta}(t)\right] .$$

From (4.1), (4.2), and (4.3) we find that, given $\tilde{\beta}(t) = \beta(t)$, $\hat{\beta}(t)$ is normally distributed with mean and variance:

$$(4.4) \qquad E\left[\hat{\beta}(t)\right] = \beta(t)$$
$$v = 2\sigma^{2}/n\Delta^{2}$$

Notice that v does not depend on t.

The experimental result (4.3) does not represent all our information about β. Even before doing the experiment we had the information developed in previous experiments. The information will be summarized in a prior distribution for $\tilde{\beta}(t)$. This distribution will be taken to be normal with

$E'\left[\tilde{\beta}(t)\right]$ = mean of prior distribution of $\tilde{\beta}(t)$,

v' = variance of prior distribution of $\tilde{\beta}(t)$.

At the beginning of period t, when the promotional rate, $x(t)$, is to be set, we have only the prior distribution. At the end of t, the experimental results are at hand and we can construct a posterior distribution. The additional information about $\beta(t)$, however, is of no use in t even though it will be helpful in $t+1$.

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* We use V [ ] and E [ ] to denote the variance and expectation operators, respectively.

5. Decision Rules for Updating the Model and Setting the Promotion Rate

Returning to the control cycle, suppose that the experiment for period t has been implemented, the market has responded, and we have in hand the sales results. We are now ready to update the model and go on to pick a promotion rate for period t+1.

Decision rules for updating the model and setting promotion can be determined by formal decision theory arguments, but we prefer to defer these and begin intuitively. One reason is that the optimality of formally derived rules is entirely dependent on the specific assumptions of the model, some of which are rather restrictive. The form of our decision rules, however, appears reasonable (although not necessarily optimal) for a wide class of situations.

The decision rules are, first, to update the mean of our prior distribution by an exponential smoothing process and, second, to set promotion rate by a formula analogous to (2.3). Specifically, we choose a number, a, such that $0 \leq a \leq 1$ and update the mean by

$$E'\left[\tilde{\beta}(t+1)\right] = a\,E'\left[\tilde{\beta}(t)\right] + (1-a)\,\hat{\beta}(t) \;. \tag{5.1}$$

Then we choose as our promotion rate for t+1:

$$x_o(t+1) = \left(m\,E'\left[\tilde{\beta}(t+1)\right] - 1\right)\Big/2m\gamma \;. \tag{5.2}$$

Suppose the process starts at t=1. It is necessary to pick a starting value $E'\left[\tilde{\beta}(0)\right]$, but, thereafter, promotion rate is set mechanistically by the rule. Since we are dealing with an exponential smoothing process, the effect of the starting value on later operations decays exponentially with t.

Notice that the rule as stated makes no assumptions about the underlying mechanisms generating $\alpha(t)$ and $\beta(t)$ and, if we think

of γ as an arbitrary positive constant, the rule is not tied down to any specific sales response process. The rule can be applied to any situation in which the experiment of Section 4 is performed, provided that somebody is willing to pick $\underline{a}$ and $\underline{\gamma}$ plus $E'\left[\tilde{\beta}(0)\right]$ and the experimental design parameters Δ and n.

The behavior of the decision rules will be clearer if we express the resulting promotion rate somewhat differently. Let us define a quantity, $\hat{x}_o(t)$, to be called the "experimental $\hat{x}_o$" by

$$(5.3) \qquad \hat{x}_o(t) = \left[m\,\hat{\beta}(t) - 1\right] \Big/ 2m\gamma .$$

This is the promotion rate that would be best if β actually equalled the experimentally determined $\hat{\beta}$. Using (5.3), (5.2) can be rewritten:

$$(5.4) \qquad x_o(t+1) = a\, x_o(t) + (1-a)\, \hat{x}_o(t)$$

Thus the decision rules amount to using a weighted combination of last period's promotion rate and the experimental $\hat{x}_o$. If we have a tight, accurate experiment, we should use a small $\underline{a}$ and so rely mostly on the current experiment. If the accuracy of the experiment is low, a large $\underline{a}$ is appropriate. Then this year's promotion rate depends mostly on last year's, which, in turn, represents a summary of considerable past experience.

Still another way of writing the decision rule brings out more sharply the role of the sales data. Using (4.3) and (5.3) we get

$$(5.5) \qquad x_o(t+1) = x_o(t) + \left[(1-a)/2m\gamma\Delta\right]\left[m(\bar{s}_2-\bar{s}_1) - \Delta\right].$$

The quantity $\left[m(\bar{s}_2-\bar{s}_1) - \Delta\right]$ is simply the experimentally estimated net profit for increasing the promotion rate from x_1 to x_2, i.e. by

an amount Δ. If the net profit is positive, $x_o(t+1)$ is made larger than $x_o(t)$; if it is negative, $x_o(t+1)$ is made smaller. The amount of the adjustment is controlled by the constant $(1-a)/2m\gamma\Delta$. If the constant is large, the promotion rate will be sensitive to the most recent experiment; if the constant is small, insensitive.

Although an appropriate choice of constants is necessary for reasonable operation, the general form of our rule provides an adaptive control system that might be expected to work fairly well for a variety of underlying sales response mechanisms. One might expect that, if the constants were chosen with one mechanism in mind, they might work satisfactorily with other mechanisms not too different. Whether or not this is so in a specific case can be explored by simulation and sensitivity analysis.

For the sales response model being assumed we now wish to motivate the decision rules more carefully and go on to calculate values for $\underline{a}$, n, and Δ in terms of the constants of the underlying process.

6. Choosing the Smoothing Constant

The smoothing constant, $\underline{a}$, was used in (5.1) to combine new and old information about β. We shall now perform a Bayesian analysis of this problem. Equation (5.1) will come out as an approximation to an exact formula.

Consider first the problem of finding the posterior distribution of $\beta(t)$ given its prior distribution and the experimental results at t. Let

$E'[\tilde{\beta}(t)]$ = mean of prior

$\hat{\beta}(t)$ = experimental mean

$E''[\tilde{\beta}(t)]$ = mean of posterior

v' = variance of prior

v = experimental variance

v'' = variance of posterior

Since the prior of $\beta(t)$ is normal and the distribution of $\hat{\beta}(t)$ given $\beta(t)$ is normal, the posterior of $\beta(t)$ is normal and has mean and variance (see [1] p. 294-5):

$$(6.1) \qquad E''[\tilde{\beta}(t)] = [v/(v+v')]\, E'[\tilde{\beta}(t)] + [v'/(v+v')]\, \hat{\beta}(t)$$

$$(6.2) \qquad v'' = v\, v'/(v+v')$$

The process generating $\beta(t+1)$ has been specified in (3.2):

$$\tilde{\beta}(t+1) = k\, \tilde{\beta}(t) + (1-k)\beta^o + \tilde{\epsilon}_\beta(t+1) .$$

As of the beginning of t+1, we know the posterior distribution for $\beta(t)$. We also know the distribution of $\epsilon_\beta(t+1)$. Since the two random variables are independent, the prior distribution of $\beta(t+1)$ has mean and variance:

$$(6.3) \qquad E'[\tilde{\beta}(t+1)] = k\, E''[\tilde{\beta}(t)] + (1-k)\beta^o$$

$$(6.4) \qquad V'[\tilde{\beta}(t+1)] = k^2\, V''[\tilde{\beta}(t)] + \sigma_\beta^{\ 2} .$$

Furthermore, the prior of β at t+1 is normal and so the normality of β is preserved as time passes.

Substituting (6.1) into (6.3), we obtain

$$(6.5) \qquad E'[\tilde{\beta}(t+1)] = k\left\{[v/(v+v')]\, E'[\tilde{\beta}(t)] + [v'/(v+v')]\, \hat{\beta}(t)\right\} + (1-k)\beta^o .$$

If k is near one, a good approximation may be obtained by setting k=1 in the above expression. Then, if we define

$$(6.6) \qquad a = v/(v+v') ,$$

(6.5) becomes the simple exponential smoothing decision rule (5.1).

It remains to express v and v' in terms of more fundamental parameters. We first observe that v' and v" will not change with t once steady state operation is achieved. This is because σ_β^2 and v do not change with t. In steady state, (6.4) and (6.2) become

$$v' = k^2 v'' + \sigma_\beta^2$$

$$v'' = v\, v'/(v+v') \ .$$

Solving for v', we obtain

$$(6.7) \qquad v' = (1/2)[\sigma_\beta^2 - (1-k^2)v] + (1/2)\{[\sigma_\beta^2 - (1-k^2)v]^2 + 4\sigma_\beta^2 v\}^{\frac{1}{2}}$$

or, when k → 1,

$$(6.8) \qquad v' = (1/2)\sigma_\beta^2 \{1 + [1 + (4v/\sigma_\beta^2)]^{\frac{1}{2}}\}$$

We know $v = 2\sigma^2/n\Delta^2$ from (4.4). Substitution of v and v' into (6.6) gives the a in terms of σ, n, Δ, k, and σ_β . Thus we have found the smoothing constants for both (5.1) and (6.5).

7. Setting the Promotion Rate

Next we wish to justify the decision rule (5.2) for setting promotion rate. Profit rate is a random variable because sales rate is

$$(7.1) \qquad \tilde{p}(t) = m\, \tilde{s}(t) - x(t) - c$$

$$(7.2) \qquad \tilde{s}(t) = \tilde{\alpha}(t) + \tilde{\beta}(t)x(t) - \gamma x^2(t)$$

We cannot maximize true profit but choose instead to maximize expected profit. At the start of t, the company's view of expected profit in t is (simplifying the notation by suppressing t):

$$(7.3) \qquad E'[\tilde{p}] = m\{E'[\tilde{\alpha}] + E'[\tilde{\beta}]\, x - \gamma x^2\} - x - c \ .$$

The company can maximize this by setting x(t) to be

(7.4) $\quad x_o(t) = (m\, E'[\tilde{\beta}(t)] - 1)/2m\gamma$.

This is (5.2).

At this point we have justified the decision rules of Section 5 for our specific model: (5.2) in general and (5.1) to the extent of k=1 approximation. If k is not close to 1, we have an exact expression in (6.5). A more convenient form, however, is the exact version of (5.4):

(7.5) $\quad x_o(t+1) = k\, a\, x_o(t) + k(1-a)\hat{x}_o(t) + (1-k)x^o$,

where we have set $x^o = (m\, \beta^o - 1)/2m$.

8. Designing the Experiment

The next job is to pick the experimental design parameters n and Δ. These will be picked to minimize the sum of two losses: the loss incurred because we do not know β exactly and the loss incurred by trying to learn β better. The losses will be calculated relative to the profits obtainable under perfect information.

With perfect information we would choose the promotion rate:

(8.1) $\quad \tilde{x}^*(t) = [m\, \tilde{\beta}(t) - 1]/2m\gamma$.

Instead we choose

(8.2) $\quad x_o(t) = [m\, E'[\tilde{\beta}(t)] - 1]/2m\gamma$.

Notice that

(8.3) $\quad E'[\tilde{x}^*(t)] = x_o(t)$.

The loss rate compared to perfect information is seen from (2.4) to be

$$\tilde{\ell}(t) = m\gamma\left[x_o(t) - \tilde{x}^*(t)\right]^2 .$$

Therefore,

$$E'[\tilde{\ell}(t)] = m\gamma V'[\tilde{x}^*(t)] = (m/4\gamma)V'[\tilde{\beta}(t)]$$

or

$$(8.4) \qquad E'[\tilde{\ell}] = (m/4\gamma)v' \ ,$$

independent of t. This is the expected loss rate relative to perfect information for those markets where we follow the decision rule to use $x_o(t)$.

In the 2n experimental markets, the expected loss rate is higher because the promotion rate is deliberately set to be different from the best available value, $x_o(t)$. Suppressing t for the moment, the experimental promotion rates are

$$x_1 = x_o - (1/2)\Delta$$

$$x_2 = x_o + (1/2)\Delta \ .$$

Consider a market at x_1. Let its loss rate relative to perfect information be $\tilde{\ell}_1$.

$$\tilde{\ell}_1 = m\gamma[x_1 - \tilde{x}^*]^2 = m\gamma[(x_o - \tilde{x}^*)^2 - \Delta(x_o - \tilde{x}^*) + (\Delta^2/4)] \ .$$

Therefore, using (8.3) and (8.4),

$$(8.5a) \qquad E'[\tilde{\ell}_1] = E[\tilde{\ell}] + m\gamma\Delta^2/4 \ .$$

Similarly, let $\tilde{\ell}_2$ be the loss rate relative to perfect information for a market at x_2. We find that

$$(8.5b) \qquad E'[\tilde{\ell}_2] = E'[\tilde{\ell}_1]$$

The total expected loss rate can now be computed. The loss rates above apply to individual markets and have the dimensions dol./hh.yr.

Let

N = total number of markets in the country,

$2n$ = the number of experimental markets,

P = the average number of households in a market,

T = total expected loss rate (dol./yr.)

We shall assume, for simplicity, that all markets have the same size, although this is not essential. Then

$$T = (N-2n)\ P\ E[\tilde{\ell}] + n\ P\ E[\tilde{\ell}_1] + n\ P\ E[\check{\ell}_2] \qquad \text{(dol./yr.)}$$

$$= N\ P\ m\ v'/4\gamma + (1/2)\ P\ m\gamma n\Delta^2$$

In later discussion of numerical results, it will be convenient to express loss rates in percentage terms. A problem arises because most quantities that are candidates for forming the denominators of such percentages are fluctuating with time. However, for several quantities we can construct reference values about which actual values fluctuate. Let

β^o = long run average of $\beta(t)$

α^o = long run average of $\alpha(t)$

$$(8.6) \qquad x^o = (m\ \beta^o - 1)/2m\gamma$$

$$(8.7) \qquad s^o = \alpha^o + \beta^o x^o - \gamma(x^o)^2 \ .$$

Here x^o is the optimal promotion rate if $\beta = \beta^o$, and s^o is the corresponding sales rate when $\alpha = \alpha^o$. These quantities make convenient reference points.

Now, let

L = expected loss rate relative to perfect information (and no experiment) as a fraction of the long run average promotion rate, x^o. (dimensionless)

(8.8) $$L = T/N\,P\,x^o = m\,v'/4\gamma x^o + m\gamma n\Delta^2/2\,N\,x^o$$

Since the right hand side is independent of t, L is the steady state expected loss.

The experimental design parameters n and Δ will be picked to minimize L. First, we observe from (8.8), (6.7), and (4.4) that n and Δ always appear in L in the combination $n\Delta^2$. We shall therefore find $n\Delta^2$ to minimize L. Then we can more or less trade off n against Δ any way we wish as long as $n\Delta^2$ is kept to its minimizing value. Practically, there are limitations (e.g., x_1 must be non-negative and 2n not greater than N, to cite two extremes) but the flexibility implied is interesting and valuable.

Finding $n\Delta^2$ to minimize L is a straight-forward, if tedious, job. In the case of the k=1 approximation the expressions are simpler and some further algebraic manipulations are worthwhile. For this case we minimize with respect to the dimensionless quantity, z, defined by means of the expression

(8.9) $$n\Delta^2 = 8\sigma^2/\sigma_\beta^2 z \text{ .}$$

Substituting (8.9) into (4.4) to get v, v into (6.8) to get v', and then v' and $n\Delta^2$ into (8.8), we obtain L in terms of z:

(8.10) $$L = (m\sigma_\beta^2/8\gamma x^o)\left[1 + (1+z)^{\frac{1}{2}}\right] + 8m\gamma\sigma^2/N\,\sigma_\beta^2\,x^o z$$

Setting dL/dz = 0, we obtain an equation that optimal z must satisfy:

(8.11) $$z/(1+z)^{\frac{1}{4}} = 8\gamma\sigma/\sigma_\beta^2\sqrt{N} \text{ .}$$

The equation can be solved for z graphically, or simply by trial and error. Therefore, given the system constants, z can be determined

from (8.11) and the optimal experimental design constant, $n\Delta^2$, can be found from (8.9).

9. Numerical Example

The behavior of the system will next be illustrated by a numerical example. Given a set of values for the constants, we design the experiment, determine the decision rules for setting promotion rate, and simulate system operation. Average losses can be calculated from the simulation or directly from expected loss formulas. In this section we suppose that the underlying model on which we have based the system design is correct. We compare optimal operation with various other policies.

The emphasis in the example will be placed on an adaptive system constructed from the k=1 approximation. One reason for this is that the approximation is fairly good. The average loss rate differs only slightly from that of the exactly optimal system. A more important reason arises in our basic view of the adaptive system. The hope is not so much for optimality with respect to a specific model as for satisfactory performance over a class of models. The simpler the decision rules, the better. When k=1, the constants k and β^o drop out. Therefore, they would not need to be estimated in setting up the system. In Section 10 we shall show that, at least in the present example, the simple rules work well even when the underlying model of market behavior is changed in various ways.

Constants. In constructing the example we have tried to pick realistic values for the constants. We have not, however, tried to represent any specific product. Sales and other figures will be given in absolute units, but the example is constructed from assumptions

about percentages. Therefore, to the extent that the results are realistic, they are intended to be realistic under changes of scale that multiply sales and promotion by the same factor.

The constants have been chosen to give sales of about 25 million dollars/year. Promotion rate is about 1.5 million dollars/year or roughly 6% of sales. Gross profit margin has been taken as 1/3 of the selling price.

The performance of the system is sensitive to the accuracy of the experiment, which in turn depends on the variance of sales among markets. We have chosen $\sigma/s^o = 7\%$, a value that has been achieved by some companies in field experiments.* We permit an experimental deviation in promotion rate of ±25% of the long term average, x^o. Then $\Delta = .5x^o$. Much large deviations have been used in practice for single experiments. The proposed deviation seems workable for continuous use.

The hardest question in picking constants is how to set reasonable values for the sales response parameters. There have been a few sales experiments that have given an indication of diminishing returns for the case of advertising. As might be expected, the results show considerable variation. We have chosen β^o and γ so that they lie within the rather wide range of values that are consistent with these experiments. The constant σ_β determines the period to period variance in sales response. The choice here has been quite subjective. However,

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* Field experiments are usually analyzed by regressions that take into account as many relevant variables as possible. The σ here refers to the residual standard deviation.

it can be given a concrete interpretation as follows: If promotion rate is held at its reference value, x^o, there will be sales fluctuations (arising solely from fluctuations in the sales response parameter β) that are normally distributed and have a standard deviation of 3% of s^o. The constant k has been taken as .9 . If k had been set at 1.0, sales response in one period would be made up of last period's response plus a random fluctuation of the type just discussed. A value of .9 gives something fairly close to this but provides a tendency for sales to move toward the reference value, s^o . Specifically, in the absence of new fluctuations, the difference between actual sales and the reference value would be reduced by 10% in each time period.

To summarize and add detail:

α^o = .32 = sales rate in absence of any promotion. (dol./hh.yr.)

β^o = 9 = long run average of sales response parameter β. (dimensionless)

γ = 100 = curvature parameter of sales response function. $(\text{dol./hh.yr})^{-1}$

m = 1/3 = gross profit as a fraction of selling price. (dimensionless)

These lead to reference values, calculated from (8.6) and (8.7):

x^o = .03 = reference promotion rate. (dol./hh.yr.)

s^o = .50 = reference sales rate. (dol./hh.yr.)

$x^o/s^o = 6\%$.

The model of sales response fluctuations has parameters:

σ_β = .5 = period to period standard deviation of $\beta(t)$. (dimensionless)

k = .9 = persistence constant for $\beta(t)$. (dimensionless)

The data related to experimental design are:

$\sigma = .07s^o = .035$ = standard deviation of sales rate for an individual market. (dol./hh.yr.)

$\Delta = .50x^o = .015$ = range of experimental deviation in promotion rate. (dol./hh.yr.)

$N = 1000$ = number of individual markets of "average" size required to make up national sales. The number of households nationally will be taken to be 50 million.

Experiment To design the experiment under the k=1 approximation, the appropriate constants are substituted in (8.11), which can be solved to give $z = 5.70$, then by (8.9), $n\Delta^2 = .00687$. Since we have arbitrarily fixed Δ at .015, n becomes 30.5 or, rounding,

$n = 30$ markets.

This completes the experimental design. In each time period 30 markets will be run at a promotion rate $.25x^o$ above the national rate, $x^o(t)$, and another 30 markets will be run the same amount below.

The resulting experimental standard error of β is calculated from (4.4) to be .602. This does not make a particularly accurate experiment, considering that we would like to operate on the sales response curve at the point with slope $1/m = 3.0$. Under the above standard error, we could fairly easily get a measured slope of 2.9 while we were actually operating at 3.5. However, two factors are present that make the situation less serious than it might appear. First of all, the current experiment does not represent all the available information about sales response. Before setting the promotion rate, current information is combined with past information by the decision rule. Secondly, profit maximizations of the present type have the property that, as long as the control variables can be kept fairly close to their optimal values, losses will be small.

Decision Rule. In order to set up the decision rule, we need the smoothing constant a. First v' is found from (6.7) to be .451. Then (6.6) gives

$$a = .446$$

and $(1-a) = .554$. This means we shall be giving fairly equal weight to new and old information. By combining (5.3) and (5.4), we can write the decision rule for $x_o(t)$ as

$$x_o(t) = a\, x_o(t-1) + \left[(1-a)/2\gamma\right]\left[\hat{\beta}(t-1) - (1/m)\right]$$

or, substituting numbers,

$$(9.1) \qquad x_o(t) = .446\, x_o(t-1) + .00277\left[\hat{\beta}(t-1) - 3.0\right].$$

We are now set to operate. Given a starting value, $x_o(0)$, and a sequence of experimental results, $\hat{\beta}(t)$, we can generate the promotion rate $x_o(t)$.

Simulation: To see how the system behaves, we have simulated the underlying market over time and have operated with the above rule. The steps in the simulation are briefly as follows: The values for k and β^o are put into (3.2) to give:

$$(9.2) \qquad \tilde{\beta}(t) = .9\, \tilde{\beta}(t-1) + .9 + \tilde{\epsilon}_\beta(t).$$

Then random numbers, $\epsilon_\beta(t)$, having mean zero and variance $\sigma_\beta^2 = 1/4$, are substituted into (9.2) to generate a sequence of values for $\beta(t)$. For the $\alpha(t)$ process we take $\alpha(t) = \alpha^o = .32$ dol./hh.yr. for all t. A varying of $\alpha(t)$ might be more realistic but, since $\alpha(t)$ does not enter into any decisions, we have simply made it a constant.

Given $\beta(t)$, the experimental results at t are simulated by

$$\hat{\beta}(t) = \beta(t) + \epsilon_{ex}(t).$$

Here $\epsilon_{ex}(t)$ is a random normal number with mean zero and variance $v = .363$. (We could simulate each of the 2n test markets separately to generate $\hat{\beta}(t)$, but it is equivalent and much easier to simulate the final experimental result directly.)

The company uses $\beta(t)$ to generate $x_o(t)$. The final sales outcome can be calculated from this, $\beta(t)$, α, and γ:

$$s(t) = .32 + \beta(t)\, x_o(t) - 100 \left[x_o(t)\right]^2 \text{ dol./hh.yr.}$$

The sales, in turn, can be used to calculate profit. Our principal criterion for judging the system, however, is the loss relative to the profit that could be made under perfect information. The best promotion rate under perfect information can be developed as a side calculation using (8.1).

$$x^*(t) = \left[\beta(t) - 3\right]/200 \text{ dol./hh.yr.}$$

The loss rate relative to this is, from (2.4):

$$\ell(t) = 33.3 \left[x_o(t) - x^*(t)\right] \text{ dol./hh.yr.}$$

The further loss resulting from the experimental deviations has not been simulated, but its expected value can be calculated from (8.5).

The simulation results are shown in Figure 5. 40 time periods are shown. (The series was started with $x(0) = x^o$ and $\beta(0) = \beta^o$ and run for 10 periods before the present data were taken.) Plotted are $\beta(t)$, which is driving the system, $x^o(t)$, by which the company responds, and the resulting $s(t)$ and $\ell(t)$. The latter is expressed as a % of x^o.

We see that the response of the adaptive system is quite good. Notice that responses in $x(t)$ lag changes in $\beta(t)$ by a time period since the information gained during one period is not available until the next. The losses are generally small, although occasional peaks

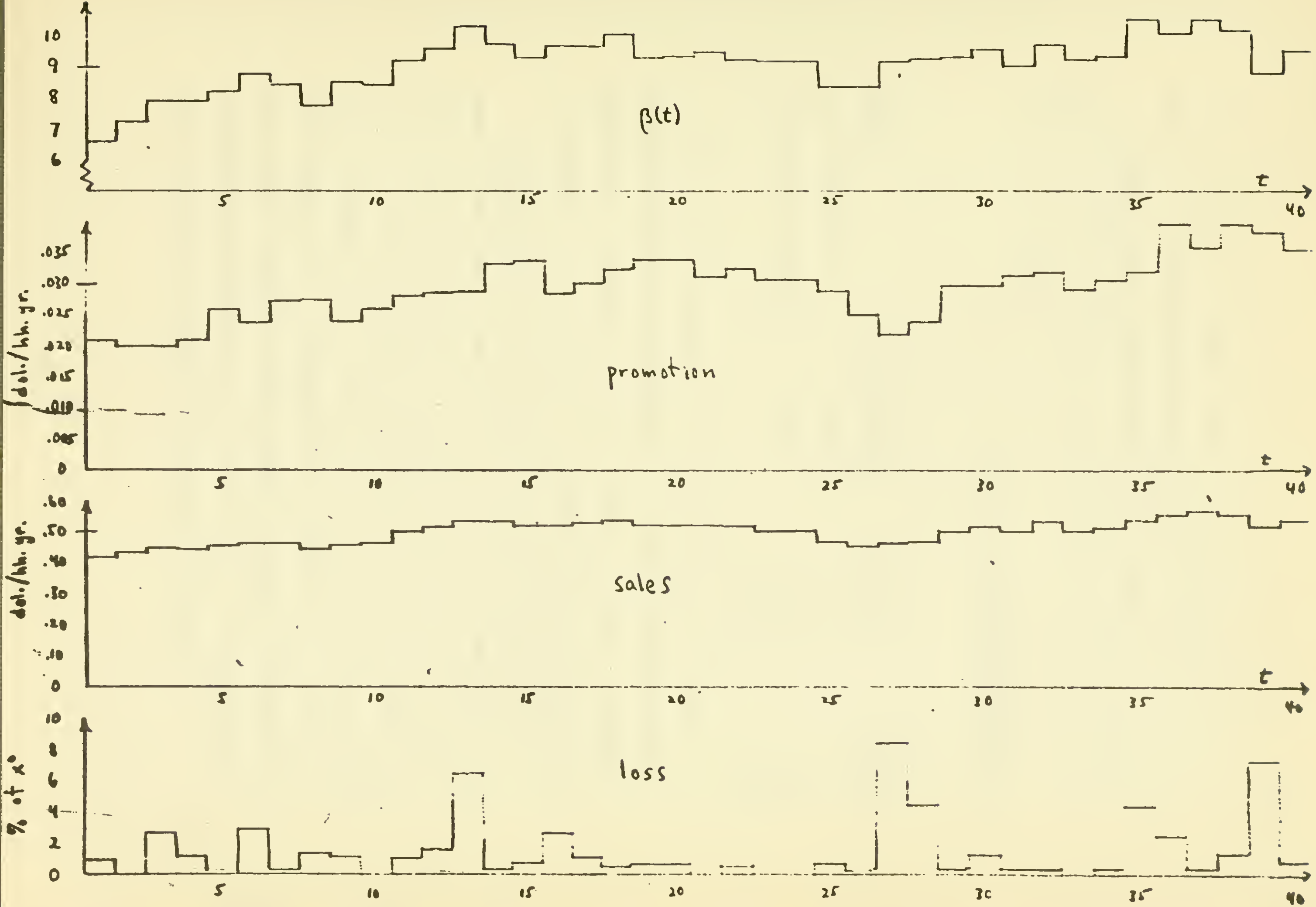

Figure 5 Simulation of adaptive control

occur where $\beta(t)$ has changed substantially and $x(t)$ has not yet caught up. The 40 period average loss is 1.55% of x^o.

Expected losses. The expected losses relative to an operation based on perfect information can be calculated from the loss formula (A.20) in the Appendix.

Expected Loss Rate
(compared with perfect information)

Source	single market (dol./hh.yr.)	national (dol./yr.)	national (% of x^o)
$x_o(t)$ not perfect	.000376	18,500	1.23
experimental deviations	.001875	5,700	.38
Total		24,200	1.61%

The 40 period average loss rate of 1.55% in the simulation is reasonably close to the calculated 1.23%. These losses are encouragingly small, especially since the standard is perfect knowledge of the response curve and since our experiment is not terribly precise.

Comparison with other policies: The results using the k=1 adaptive system may be compared with other types of operations. One possibility is to set $x(t)$ to a constant value, say x^o, for all t. The rate x^o has the property of being optimal when $\beta(t)$ takes on its long run average value. It is not clear, of course, how the company would figure out x^o. Consequently the values $.5x^o$, x^o, and $1.5x^o$ will be tried. Expected losses compared to perfect information can be computed with the help of (2.4) and (A.10). For a constant promotion rate there is no cost of experimental deviations.

	Expected Loss Rate (compared to perfect information)	
	(dol.yr.)	(% of x^o)
adaptive system: $x_o(t)$	24,200	1.61%
constant rate: x^o	54,700	3.65
constant rate: $.5x^o$	430,000	28.65
constant rate: $1.5x^o$	430,000	28.65

Thus the increased loss ranges from 2% of x^o for the lucky guess to 27% of x^o for the substantial deviation.

Suppose that circumstances prevent an experiment of the proposed size. Instead of 30 markets in each group, suppose we have to get along with 15. Then the experimental error would increase from .602 to .726. The value of a would decrease from .561 to .446, implying more reliance on past information. The system, however, is still adaptive.

	Expected Loss Rate (compared to perfect information)	
	(dol.yr.)	(% of x^o)
proposed experiment (n=30)	24,200	1.61%
smaller experiment (n=15)	25,500	1.70

The deterioration in performance is small.

Finally, the k=1 system is compared with the optimal system based on the true value k=.9 . The latter system uses the decision rule (7.5).

	Expected Loss Rate (compared to perfect information) (dol.yr.)	 (% of x^o)
adaptive system: k=1	24,200	1.61%
optimal adaptive system	22,300	1.49%

The expected loss in the optimal system is close to that obtained by the k=1 approximation. The optimal experiment is somewhat smaller, n=22, and the smoothing constant somewhat larger, a=.53,.

10. Sensitivity Analysis

The k=1 adaptive system performed well in the example of the previous section, but the near optimality of the system is based on various assumptions about the underlying operation of the market. Many of these assumptions are questionable. How will the system perform if some of them are incorrect? To investigate this question,

we use the experiment and decision rule as derived but change the underlying market model in various ways.

Constant Sales Response. The original model assumed that sales response fluctuates over time. Suppose that, instead, sales response is constant and has $\beta(t) = \beta^o$. Then the optimal promotion rate is $x(t) = x^o$. If we use the adaptive system of the previous section some relative loss will be incurred. The loss can be calculated using the formulas in the appendix by setting $\sigma_\beta = 0$.

	Expected Loss Rate (compared to perfect information) (dol./yr.)	 (% of x^o)
adaptive system	11,000	.74
constant rate: x^o	0	0
constant rate: $.5x^o$	375,000	25.
constant rate: $1.5x^o$	375,000	25.

We see that the average loss rate using the adaptive system is only .74% of x^o. This is quite small; in fact, it is less than the 1.61% that occurred when sales response fluctuated. (The reason is that the stable sales response is inherently easier to optimize.) Of course, if the company president is clairvoyant, he will pick $x(t) = x^o$ and have no loss at all. On the other hand, if he misses and sets $x(t) = .5x^o$ or $1.5x^o$, a substantial loss is incurred.

Changes in Sales Response Have No Persistence. The earlier model assumed that sales response changed but that the starting point for the change was the previous sales response. Thus a goal of the adaptive system was to follow sales response as it drifted about. Suppose instead that sales is subject to fluctuation but has no persistence.

In other words, $\beta(t)$ equals β^o plus an independent random variable for each t. This situation may be obtained in our model by setting $k = 0$. Under these circumstances the optimal policy is $x(t) = x^o$.

	Expected Loss Rate (compared to perfect information) (dol./yr.)	($\%$ of x^o)
adaptive system	25,800	1.72
constant rate: x^o	10,400	.70
constant rate: $.5x^o$	386,000	25.7
constant rate: $1.5x^o$	386,000	25.7

The adaptive system is not as good as the constant rate x^o but the difference is small. On the other hand if a constant rate substantially different from x^o is picked, large losses are incurred.

Different curvature. Perhaps the most serious assumption in the model is that γ, the curvature parameter in sales response, is a known constant. In practice, γ will usually be unknown or poorly known. We suggested earlier that perhaps our ignorance of γ was not too serious. To test this possibility we make some fairly drastic changes in ' but set promotion rate by the adaptive system derived from the old value.

First, γ is reduced from 100 to 25, a factor of 4. Sales response is now much more linear.

	Expected Loss Rate (compared to perfect information) (dol./yr.)	($\%$ of x^o)
adaptive system	75,000	5.01
constant rate: x^o	219,000	14.6
constant rate: $.5x^o$	312,000	20.6
constant rate: $1.5x^o$	312,000	20.6

The losses compared to perfect information are found to be higher than they were for $\gamma = 100$ for both the adaptive system and the constant rate, x^o. The more linear response means that a change in β can be made the basis for a substantial and profitable change in promotion rate (if response is known perfectly). Without perfect information, both ways of operating fumble more than formerly. However, the adaptive system does much better than the best constant value.

Next we increase γ by a factor of 4 to 400. Sales response is now much more nonlinear.

	Expected Loss Rate (compared to perfect information) (dol./yr.)	 (% of x^o)
adaptive system	27,100	1.81
constant rate: x^o	13,600	.91
constant rate: $.9x^o$	73,600	4.91
constant rate: $1.1x^o$	73,600	4.91

Here a clairvoyantly picked constant rate is somewhat better than the adaptive system. However sales and profit rates are very sensitive to promotion. Constant rates of $.5x^o$ and $1.5x^o$ lead to unrealistic positions on the sales response curve and consequent large losses. Therefore, we have substituted the milder rates of $.9x^o$ and $1.1x^o$. These values, although quite close to the best constant rate are seen to perform worse than the adaptive system.

Behavior with a Step Function. Insight into the behavior of the adaptive system can be gained by seeing how it reacts to a step change in β. Figure 6a shows a $\beta(t)$ that goes along as $\beta^o = 9$ until $t = 1$ and then jumps to 15. Under perfect information optimal

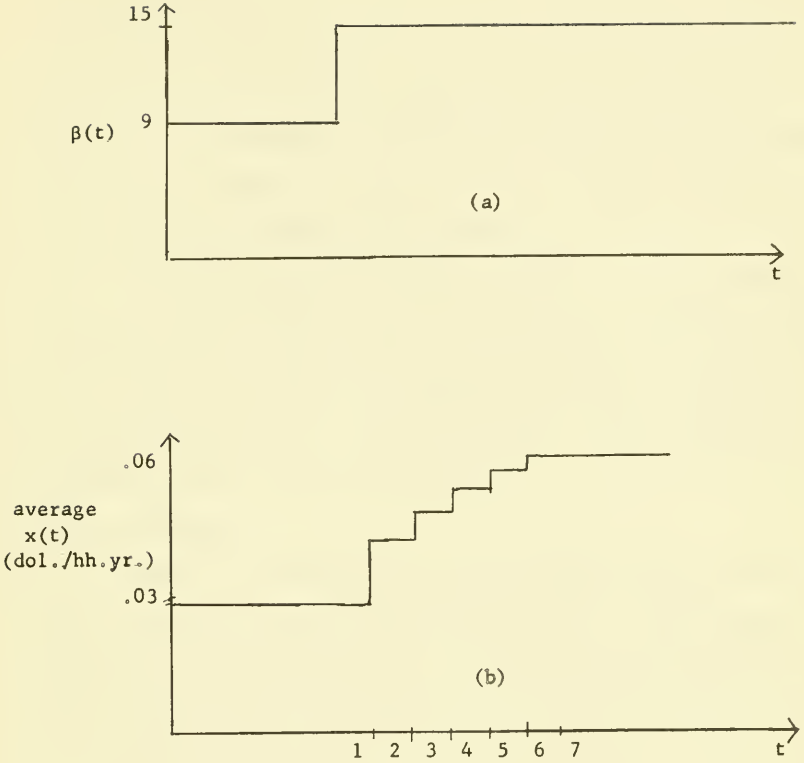

Figure 6. Behavior of average x(t) when β(t) jumps.

operation would be $x^o = .03$, jumping to .06 at $t = 1$. Figure 6b shows the expected value for $x(t)$ under the adpative system of the previous section. (Because of randomness in the experimental results, actual operation would fluctuate about the expected value.) We see that average $x(t)$ lags one time period before starting to respond and then responds fairly quickly.

In summary, we have devised sensitivity tests to see how the adaptive system performs when the assumptions on which it is based are violated in various ways. These tests have been applied to the numerical example of the previous section. The performance of the adaptive system is found to be good.

11. Discussion

Perhaps it is surprising that such simple operating rules can cope successfully with a changing environment. Other examples, however, are not hard to find. The home thermostat controls temperature under widely varying conditions of heat loss and does it without solving heat flow equations on a digital computer. In our case there are several reasons why the adaptive system operates well. In the first place, the system overcomes some of the inaccuracy of its measuring device by working from an accumulation of present and past information. In the second place, sales response is presumed (reasonably enough) to vary smoothly with spending. As a result, the profit maximum is also smooth. This means that small deviations from best operation cost very little; underspending gives fewer sales but saves almost an equal amount in out-of-pocket expense; overspending is almost counterbalanced by increased sales. Substantial losses are produced only by large deviations and these tend to

be avoided by the adaptive system. Thirdly, various empirical studies suggest that, once some nominal promotion rate is achieved, spending rate per se is not a big factor is sales. (The rapid diminishing returns of the log curve used by Benjamin and Maitland [2] can be interpreted this way.) This is not to say that increases or decreases may not be profitable but rather to say that they are not likely to cause the jumps in sales that are caused, for example, by product changes or, sometimes, by changes in promotional treatment.*

The use of the adaptive system requires a tolerance of explicit uncertainty on the part of a company's management. The system will specify some definite number for promotion rate, but the available information on response may be sufficiently ambiguous that other, rather different numbers look almost as good. Such a situation may be disconcerting. Some people prefer the pseudo-certainty of a plan that is defended as exactly right (even though something quite different was done the previous year under roughly the same circumstances.) The adaptive system must be regarded as a set of operating rules that produce a good average return rather than as a device for producing the perfect number each time. In fact, uncertainty plays an essential role in the system since inaccuracy is deliberately accepted in the optimal design.

11.1 Practical Problems. We shall try to anticipate and discuss some practical questions that might come up using a system like the one presented here. First of all, there is a class of questions relating to setting up the system: Is the model as it stands sufficiently

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* There is an extremely interesting question implicit here: If some aspect of promotional treatment, for example, advertising copy, is exceptionally good, should a company spend more or less on promotion? The answer does not seem obvious.

good or should certain complications be added? Several of the parameters of the model will almost certain be unknown; how should they be picked? The decision rules have been derived under the assumption of steady state operation; should anything different be done in starting up?

One approach to handling the uncertainties behind these questions would be to develop a formal Bayesian analysis of them. However, the approach suggested by the work here is to gather together whatever relevant information can be found, build a specific model, and investigate its behavior by simulation and sensitivity analysis. For starting up, the past promotion rate of the company could be used as the past promotion rate required by the model. It might be desirable to design extra accuracy into the first few experiments. As a practical matter, however, experimental accuracy will probably increase rather than decrease with time because of increasing experience.

Over a number of time periods, information can be built up on the validity of the model and better estimates can be made of the constants. Particularly important is information about γ, since γ expresses the degree of diminishing returns. Notice that our proposed operation collects information about γ even though we have not acknowledged this fact in the analysis. The information is the result of using three spending levels $(x_o - (\Delta/2), x_o, (x_o + (\Delta/2))$. These permit an estimate of γ in each time period. Individual estimates are likely to be quite unreliable, however. This is one reason we have chosen not to build the measurement of γ directly into the analysis.

Another practical problem is that the decision rules may occasionally call for a really substantial change in spending rate. In

some cases the proposed rate may fall outside the range included in the experiment. Substantial changes are often disturbing to an organization and changes beyond the experimental conditions are on less solid empirical ground than those within. One way to handle the situation is by exercise of the managerial override that obviously exists on the whole system. A better way, however, and one that can be pre-planned and pre-studied is to clamp the amount of change permitted, say, by limiting it to ±15% or ±20%. This will tend to slow down system response, but perhaps not excessively so.

Different market areas, it may be argued, will have different sales responses to promotion, whereas the measurements discussed here produce an average response. It seems certainly true that markets will differ, at least to some degree. However,whether the differences are appreciable will depend on the situation. Where they are, it would be desirable to take advantage of them. Possibly, one can develop an adaptive system that applies to individual markets, but the measurement problems appear to be quite difficult. A feasible approach would be to use whatever empirical and judgmental information is available to develop individual market adjustments to apply to the average curve. In this case each market would have its own $x_o(t)$ and experimental deviations would be made relative to this.

A related problem concerns the nature of the test markets. Assuming, as we generally have, that the basic experimental unit is a market area, the test markets themselves will usually be medium-sized markets. The very large and very small tend to be ruled out by various operating considerations. Yet, we wish to set promotional

rate throughout the country. This is basically a problem of individual market adjustments and can be handled as discussed above.

The experimental design discussion of Section 8 omits a cost that may sometimes be appreciable, namely, the out-of-pocket cost of running the experiment. If the cost is substantial, it can easily be included in the analysis for determining the optimal experiment. If there is a cost proportional to the number of test markets, the quantity $n\Delta^2$ will no longer be an invariant constant. Instead, the calculation will show that total cost is least when the number of markets, n, is as small as possible and the experimental deviation, Δ, in the promotion rate is as large as possible. We do not want Δ to become so large that sales are substantially reduced in the low markets. Although this consideration could be included in the formal analysis, a simpler procedure is to place an arbitrary upper limit on Δ. Then n can be calculated in a straightforward way.

The effect of promotion may be delayed. Empirical data (see references 3, 4, 5, and 6) suggest that the principal response is frequently fairly rapid, say, within one to three months. If sales are measured by factory shipments, a pipeline delay will also be encountered. A nominal value might to 1-1/2 months. In the discussion here we have obviously assumed that delays are small enough that a measurement of the promotional effort can be made within the experimental time period. A period of a year seems appropriate for many situations. A year also eliminates certain problems of seasonality and fits in with the budgeting process.

Any control system should be examined for stability. Conceivably, an inappropriate choice of constants relative to the underlying process could result in too small a value of the smoothing

constant, a, and so an inappropriately large dependence on the most recent data. This could perhaps lead to an oscillation of underspending alternating with overspending However, this does not seem likely for the type of decision rule being used. In any case, suspected sources of oscillation can be investigated in advance by simulation.

11.2 Competition. Businessmen frequently are concerned about a possible self-defeating aspect in promotional competition. If the effect of promotion is primarily on market share and not on total demand and, if promotional efficiency is comparable from one company to another in the industry, then promotional increases may appear profitable in experiments. Yet, when the increases are applied nationally, they may be countered by competing companies in such a way that nobody's sales change much but everybody's spending is increased. Models of this sort of process have been built by a number of writers. Mills'[8] paper contains an example.

Several remarks can be made. Let us consider the extreme case where promotion affects only market share and not total industry sales. If the companies are operating at competitive equilibrium, i.e., the companies are already individually operating (within their measurement capabilities) in the neighborhood of their independent maximum profit points, then the adaptive system will continue this type of operation in an efficient manner. Whenever a company starts over- or under-spending, the adaptive system will tend to return the spending rate to the maximum profit position.

If, on the other hand, all companies are spending less than they would at competitive equilibrium, i.e., an accurate experiment would

indicate that increases would be profitable for each company if the others stayed constant, then spending rates tend to be unstable. A company not wishing to disturb the situation may choose to hold promotional spending constant or to adopt a figure close to the industry average spending as a per cent of sales. Either way, there is no need for an adaptive system to set promotional spending for it is already set. (There may be some other interesting applications, however, in the allocation of funds between promotional alternatives.)

Another policy for this situation, but one that is more likely to be misunderstood by the competition, is to set stiffer return requirements on promotional spending than those implied by the conventional maximum profit calculation. This could be done by using a smaller value for the gross margin than actually was the case.

It is a rare company that knows whether it is in competitive equilibrium with respect to promotional spending and whether its spending appreciably affects industry sales. Some information about these questions can be obtained in experiments of the type we have been considering, although, as we have discussed, the information is likely to deterioriate unless kept up to date.

11.3 Extensions. We mention three particularly desirable extensions of the model. First it would be helpful to have an adaptive system to allocate a fixed budget between several promotional alternatives. Second, it is unrealistic and undesirable to rely solely on past experiments to estimate sales response. Certain other pertinent information is usually available; knowledge of product changes, forecasts of economic conditions, etc. A step could be introduced into the feedback loop to bring this information into the

prior distributions used to set promotional rate. Finally, it would be worthwhile to study the possibility of monitoring individual market response by time series analysis as a basis for individual market adjustments.

12. Conclusions

The adaptive system discussed here is directed toward a major continuing problem: the setting of spending rate for promotion. The concept of the adaptive system seems basically correct: a company should learn from experience in an organized way. The model studied is a simple one but it may be useful as it stands and it certainly is capable of extension. The operating rules that have been developed are simple, feasible, and seem intuitively reasonable. In examples using realtistic numbers, system performance has been good, despite a relatively inaccurate measurement process. Of particular interest is the insensitivity of performance to substantial changes in the underlying model..

Appendix

The mathematical development in the body of the paper was directed toward finding optimal operation. However, for sensitivity analysis it is helpful to have exact expressions for the expected loss rate under various kinds of non-optimal operation. Accordingly we here solve the following problem:

Given (1) the basic environmental models

$$s = \alpha + \beta(t)x - \gamma x^2 \tag{A.1}$$

$$\beta(t) = k\,\beta(t-1) + (1-k)\beta^o + \epsilon_\beta(t) \tag{A.2}$$

(2) the decision rules

$$x_o(t+1) = a\,x_o(t) + (1-a)\hat{x}_o(t) \tag{A.3}$$

$$\hat{x}_o(t) = [m\,\hat{\beta}(t) - 1]/2m\gamma \tag{A.4}$$

(3) the stochastic process describing the experiment

$$\hat{\beta}(t) = \beta(t) + \epsilon_{ex}(t) \tag{A.5}$$

(4) the loss rate formulas

$$\ell(t) = m\gamma[x_o(t) - x^*(t)]^2 \tag{A.6a}$$

$$\ell_i(t) = m\gamma[x_i(t) - x^*(t)]^2 \quad , \quad i=1,2 \quad \text{where} \tag{A.6b}$$

$$x^*(t) = [m\,\beta(t) - 1]/2m\gamma \; ; \tag{A.6c}$$

$$L = \{(N-2n)P\,E[\ell] + n\,P\,E[\ell_1] + n\,P\,E[\ell_2]\}/N\,P\,x^o \tag{A.6d}$$

Find the steady state expected loss rate, L, in terms of the underlying constants of the model.

We shall do this for an arbitrary (not optimal) smoothing constant, *a*, and an arbitrary (not optimal) experiment of accuracy $V(\epsilon_{ex}) = \sigma_{ex}^2 = 2\sigma^2/n\Delta^2$. The calculation will proceed by solving different equations to express $\beta(t)$, $x^*(t)$, and $x_o(t)$ as infinite sums of independent random

variables. The desired steady state expected loss can be calculated from these.

Successive substitutions of (A.2) into itself give

$$\beta(t) = \beta^{o} + \sum_{j=0}^{\infty} k^{j} \epsilon_{\beta}(t-j), \tag{A.7}$$

so that in steady state

$$E[\beta(t)] = \beta^{o} \tag{A.8}$$

$$V[\beta(t)] = \sigma_{\beta}^{2}/(1-k)^{2} . \tag{A.9}$$

Putting (A.7) into (A.6c) gives

$$x^{*}(t) = x^{o} + \frac{1}{2\gamma} \sum_{j=0}^{\infty} k^{j} \epsilon_{\beta}(t-j) \tag{A.10}$$

Therefore in steady state

$$E[x^{*}(t)] = x^{o} \tag{A.11}$$

$$V[x^{*}(t)] = (1/4\gamma^{2})\, \sigma_{\beta}^{2}/(1-k^{2}) \tag{A.12}$$

The calculation of $x_{o}(t)$ takes a little longer. Successive substitutions of (A.3) into itself give

$$x_{o}(t+1) = (1-a) \sum_{j=0}^{\infty} a^{j}\, \hat{x}_{o}(t-j) \tag{A.13}$$

From (A.5) and (A.7) we have

$$\hat{\beta}(t) = \beta^{o} + \sum_{j=0}^{\infty} k^{j} \epsilon_{\beta}(t-j) + \epsilon_{ex}(t),$$

so that, from (A.4)

$$\hat{x}_{o}(t) = x_{o} + (1/2\gamma) \sum_{j=0}^{\infty} k^{j} \epsilon_{\beta}(t-j) + [\epsilon_{ex}(t)]/2\gamma .$$

Putting this in (A.13), we finally obtain

(A.14)
$$x_o(t+1) = x^o + \left[(1-a)/2\gamma(k-a)\right] \sum_{s=0}^{\infty} (k^{s+1} - a^{s+1})\epsilon_\beta(t-s) + \left[(1-a)/2\gamma\right] \sum_{j=0}^{\infty} a^j \epsilon_{ex}(t-j) \ .$$

In steady state, therefore,

(A.15)
$$E\left[x_o(t+1)\right] = x^o$$

(A.16)
$$V\left[x_o(t+1)\right] = \left[(1-a)/4\gamma^2(1+a)\right]\left[\ \sigma_\beta{}^2(1+ak)/(1-ak)(1-k^2) + \sigma_{ex}{}^2\right.$$

Some manipulation using (A.14) and (A.10) gives the following result to be used shortly:

(A.17)
$$E\left[\{x_o(t+1) - x^o\}\{x^*(t+1) - x^o\}\right] = \left[\sigma_\beta{}^2/4\gamma^2\right]\left[k(1-a)/(1-k^2)(1-ak)\right].$$

Turning now to the loss rates, we observe that (A.6a) can be written:

$$\ell(t) = m\gamma\left\{\left[x_o(t) - x^o\right]^2 - 2\left[x_o(t) - x^o\right]\left[x^*(t) - x^o\right] + \left[x^*(t) - x^o\right]^2\right\},$$

so that
$$E\left[\ell(t)\right] = m\gamma\left\{V\left[x_o(t)\right] + V\left[x^*(t)\right] - 2\,E\left[\{x_o(t) - x^o\}\{x^*(t) - x^o\}\right]\right\}.$$

Using now (A.16), (A.12) and (A.17), we have

(A.18)
$$E\left[\ell(t)\right] = \left[m/4\gamma\right]\left[(1-a)\sigma_{ex}{}^2/(1+a) + 2\sigma_\beta{}^2/(1+k)(1+a)(1-ak)\right]$$

In the case of the experimental markets the analysis of Section 8 holds so that

$$E[\ell_1(t)] = E[\ell_2(t)] = E[\ell(t)] + m\gamma\Delta^2/4 \,. \tag{A.19}$$

Substituting (A.18) and (A.19) into (A.6d), we obtain the desired quantity:

$$L = [m/4\gamma x^o]\{[(1-a)/(1+a)][2\sigma^2/n\Delta^2] + 2\sigma_\beta^2/(1+k)(1+a)(1-ak)\} + m\gamma n\Delta^2/2Nx^o \tag{A.20}$$

Finally, it is of some interest to find $E[s(t)]$:

$$E[s(t)] = s^o + [(1-a)/(1+a)4\gamma][\sigma_\beta^2(ak+2k-1)/(1-ak)(1-k^2) - \sigma_{ex}^2] \tag{A.21}$$

Notice that only under special circumstances will $E[s(t)] = s^o$.

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